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EDITORIAL

A Cycle Primer

THIS issue of the Journal of Cycle Research carries a paper on cycles in Lynx abundance. It has been written with two purposes in mind: (a) to report on a partial study of Lynx cycles and (b) to furnish a "primer" of analytical methods that others may find useful in studying their own data for cycles. Instead of Lynx fur returns, other figures may be substituted, yet the procedures for analysis will be substantially the same. An investment banker could use stock market prices, and a commodity broker, the prices of commodities. A corporation manager could use the sales or production of his industrial concern. An agricultural economist would find it worth while to use crop acerages or crop yields. A meteorologist could use weather data, a hydraulic engineer the run-off of rivers, and a geologist the records of varves. A medical statistican might use disease reports and a game biologist hunting records. The measurements of tree rings could be used by foresters. And a radio engineer could use interference with radio transmission. Thus, the data could be of any manifestation of man or nature wherein cycles may be present or presumed to be present. Though a reader may have no interest in the Lynx itself, one interested in analysing records for cycles should find the paper useful as an introduction to cycle analysis.

CYCLES OF LYNX ABUNDANCE

BY LEONARD W. WING

AN index of Lynx *(Lynx canadensis)* abundance has a rhythm with wave length varying from about 6 to 11 years. When subjected to cycle analysis the index shows a dominant (alpha) cycle with an average length of 9.6 years and with an average amplitude of 257.0% of trend at its high and 38.9% of trend at its lows. The cycle indicated as probably the beta cycle has an average length of 35.2 years and average amplitude of 144.5% of trend at its highs and 69.2% of trend at its lows. A cycle designated as probably the gamma cycle has an average length of 22.2 years with an average amplitude of 138.0% of trend at its highs and 72.4% of trend at its lows. Five other cycles with average lengths of 7.46, 7.95, 9.0, 11.75, and 15.05 years are isolated and measured. The analysis indicates also the presence of a number of additional cycles.

It is well-known that the abundance of Canadian Lynx fluctuates in a rhythm that from crest to crest or from trough to trough, varies from eight to eleven years in length. This variable cycle visible in the plotted data is here termed the **manifest cycle** to distinguish it from the mathematical or "pure" cycles of which it may be formed.

A rough measure of the abundance of the Lynx in Canada can be obtained from various sources, chiefly the records of offerings of Lynx skins to the Hudson's Bay Company. These various records are available from 1735 to date. An index of Lynx abundance based on these various sources of information has been prepared* and is plotted as Figure 1.

Because of a discontinuity of 1820 this index is not a suitable measure of long-term trend as between the ninety years preceding 1820, and the ninety years following. Figure 21, for example, shows that the trend before 1820 is probably too high for the succeeding years. For the purpose of cycle analysis, however, distortions from splicing the series may be overlooked for the present, as there is no evidence that they are of cyclical nature. There is also no evidence that indicates trapping or trading in a cyclic manner. But we must keep in mind the possibility that amplitude and phase of any cycles may be distorted temporarily. A high or low may be missing in some cases, yet the cycle will return in correct timing later. Many of these influences have been eliminated by use of trends and deviations from trends. Another source of weakness is a geographic one. The index combines records from across Canada, and thereby integrates the fur returns, which would tend to mask local, regional, or ecological variations.

It is important to determine the length of the Lynx cycle as accurately as possible for two reasons: (1) the more accurate the determination of wave length, the more accurate will be any forecast of Lynx abundance and the offerings of Lynx skins that may be based upon this characteristic cyclic behavior; (2) an accurate determination of average cycle length may suggest, either by association or elimination, other phenomena that could or could not be related to the cycle of Lynx abundance. If the cycle of Lynx abundance turns out to have an average wave length of 9.6 ± .05 years, for example, and the abundance of Atlantic salmon turns out to have a cycle with an average of 9.6 ± .05 years also, a common environmental cause might be the explanation of both behaviors. On the other hand if the cycle in the abundance of Atlantic salmon turns out to be 9.4 years ± .05 years, there can be no probable interrelationship or common cause for the observed behavior.

Because sunspots have a manifest cycle of eight to sixteen years in length, the obvious suggestion occurs that sunspots might in some way be the cause of the cycle in the abundance of Lynx. A few minutes' study of Figure 1, however, shows that the manifest cycle of Lynx abundance averages less than ten years in length and an equally casual study of sunspot numbers shows that this manifest cycle averages something more than eleven years in length. As any differences in wave length create a cumulative difference between crests (and troughs) as time goes on, it is obvious that there is no direct interrelationship between the two manifest cycles. But lesser Lynx cycles and lesser sunspot cycles could be interrelated, if of the same wave length. Only when the various lesser sunspot and Lynx cycles have been isolated and measured would we know.

Graphing of Cycles. In this analysis, three kinds of graph paper have been used, and the purposes of each will be explained briefly. The initial graph of the Lynx index has been done on semi-logarithmic paper. The ruling of

*WING, LEONARD W., 1953. "AN INDEX OF LYNX ABUNDANCE." JOURNAL OF CYCLE RESEARCH, 2: 21-24.

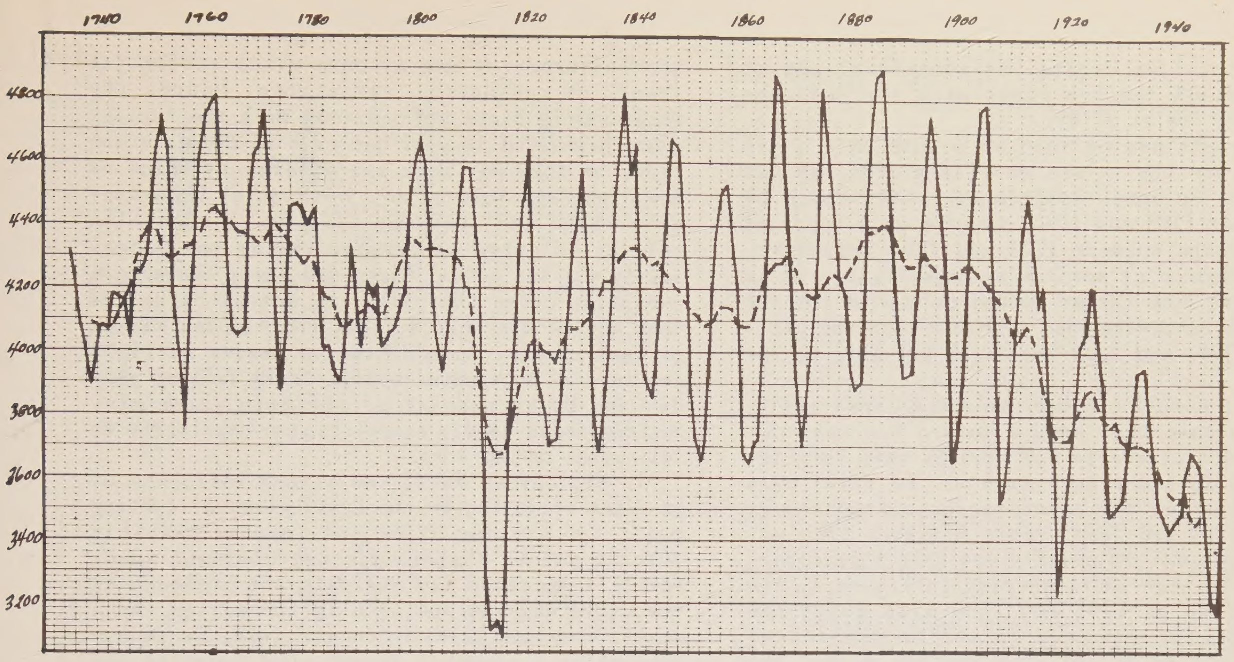


FIGURE 1. THE INDEX OF LYNX ABUNDANCE SHOWS A MANIFEST CYCLE OF ABOUT 9.6-YEARS LENGTH BUT VARYING BETWEEN SEVEN AND THIRTEEN YEARS FOR SUCCESSIVE HIGHS OR LOWS. IN THIS CURVE, THE LOGARITHMS OF THE INDEX HAVE BEEN PLOTTED ON ARITHMETIC RULING. IT SHOULD BE COMPARED WITH THE CURVE ON PAGE 22, (JOURNAL OF CYCLE RESEARCH VOL.2, NO.1) THAT USES THE INDEX AND PLOTS IT ON A SEMI-LOGARITHM RULING. THE DASHED LINE IS THE 9-YEAR MOVING AVERAGE. IN *Cycles, A Monthly Report* FOR OCTOBER 1950, E. R. DEWEY NOTED POSSIBLE CYCLES OF ABOUT 20 OR 21 AND 35 OR 36 YEARS. THESE MEASURE AT 22.2 AND 35.2 YEARS.

semi-logarithmic paper gives the graphed lines an appearance of percentage graphing, as each line of the graph varies from its predecessor by a percentage. All graphing of logarithmic data has been upon arithmetic polypurpose paper ruled twelve lines to the horizontal inch and twenty to the vertical inch (183 x 180 lines to the page) in sheets 16 1/4 x 11 inches, the same size as the semi-logarithmic paper. For fitting curves and for measuring slippage between sections of periodic tables, 8 1/2 x 11 cross section paper ruled five lines to the inch has been used. Sheets have been pasted together when necessary to make them larger.*

Use of Logarithms. Data common in nature are usually proportionate and seem to follow the "percentage rule" rather universally. Thus, if a hundred Lynxes increase to 150, the increase is 50%. A thousand increasing to fifteen hundred or a million to a million and a half would still show the same proportionate increase. This is sometimes referred to as "geometric increase." Logarithms prove easier to use than the original data, and their use materially aids cycle analysis. Data may be analysed for cycles by use of the original figures, but parodoxical though it may seem, the labor involved is much greater than when the numbers have been converted to logarithms. The Lynx index therefore has been converted to logarithms and logarithms used throughout this study (Table 1).

Determination of Wave Length of the Dominant (Alpha) Lynx Cycle. There are various ways of determining the average or typical length of a rhythm appearing in any series of figures. Probably the simplest way—and the one generally in use—is to measure the time interval between the first crest and the last crest and to divide this period by the total number of waves intervening—that is, by the total number of crests in the series less one. This method applied to irregular data has two serious drawbacks: (1) how do you determine whether a given turning point is or is not a "crest?" and (2) assuming the correct answer to the first question, how do you know but that the positions of the first and or last crest may not be distorted?

Taking the Lynx figures (Figure 1), for example, there can be little argument in regard to the existence of five crests between 1857 and 1905, inclusive, but between 1771 and 1800 inclusive one man might pick four crests (one each at 1771, 1780, 1787, and 1800). Another person

*THE SEMI-LOGARITHMIC AND ARITHMETIC PAPERS USED ARE CODEX POLYPURPOSE GRAPH PAPER MADE BY THE CODEX BOOK COMPANY, INC., NORWOOD, MASSACHUSETTS. SUITABLE CROSS SECTION PAPER FOR TIME CHARTS AND PERIODIC TABLE CURVES MAY BE OBTAINED AT MOST STATIONARY STORES.

might pick five crests (an additional one at 1790), still another might pick six (all the above plus one at 1778).

If crests are defined as points on a curve higher than the values on either side, as has been done by some, additional crests could be counted at 1783 and 1792, to give us a total of eight crests between 1771 and 1800 inclusive. As the essence of science is objectivity, it is clear that some better method is needed for the selection of crests (and troughs) than the picking of highs and lows that seem suitable to the picker.

Not to complicate our problem at this point, let us assume that such a method has been devised and that crests and troughs have been determined in accordance with it. We still have the problem of the inaccuracies introduced by using first and last crests wherever they may happen to come. An example may make this clearer. In the Lynx index charted in Figure 1, the high values of 1735 and 1950 with which the series begins and ends cannot be considered crests, for we do not know whether or not the preceeding and following values respectively (values for 1734 and 1951) were lower. We are forced then to take our first crest at 1744 and our last crest at 1944. Suppose now that we pick 21 waves (22 crests) between these two dates inclusive. Under these circumstances, we would get an average length of 9.52 years (1944-1744 = 200; 200 ÷ 21 = 9.52).

But we may well imagine that our figures could have begun one wave later in 1750 instead of in 1735. In this event our first crest would be in 1752. Picking the same intervening crests as before, we would get an average length of 9.6 years (1944 - 1752 = 188; 188 ÷ 20 = 9.6). In a series as long as this, the difference between the two average lengths is small; but in other instances where only a short series of figures is available the discrepancy can be much larger. If, for example, figures of Lynx abundance were available only from 1910, the average length of a Lynx cycle measured from the first crest (1913 to the date of the last crest (1944) is 10.3 years (1944 - 1913 = 31; 31 ÷ 3 = 10.31). If now we take ten more years of figures, the first crest would fall in 1905; applying the same method, we would obtain an average length of 9.75 years (1944 - 1905 = 39; 39 ÷ 4 = 9.75). The difference between the two determinations, .55 years, becomes decidedly important.

Locating the Position of Waves in a Series. In a very real sense, the picking of highs and lows sets the initial approach in identifying the length of cycles and their position in the series being analysed. The length of a cycle and its position in the series determines where the highs and lows *of that cycle* fall. But they may be completely masked in the final figures by the influence of one or more other cycles, or trend, or accidental variations. Hence, the recognizable highs and lows may mark the regions where highs and lows of one or more cycles coincide, or they may mark the general position of the highs and lows of an alpha cycle. Recognizable highs and lows may also be of noncyclic origin. No curve of regular rhythm may necessarily fit the actual highs and lows; conversely a regular curve that fits the actual highs and lows may be one that follows the blended effect of real cycles. In addition, a series of highs and lows of accidental origin could fall at random and be at regular or rhythmic intervals. But one would hardly expect random regularity to fall often or for long.

In biological data, the strong influence of an alpha cycle (as in the Lynx) gives a starting point for picking highs and lows as possible turning points that locate the dominant wave. Simple inspection may indicate the presence of the cycle and its position in the series. In a sense, when one picks highs and lows, he is establishing a possible length of a cycle if one is present. In discussing procedure, it is hardly possible to treat ways of picking highs and lows separate from the location of a wave in a series or from the length of cycles. Even though discussed separately, therefore, all three things must be kept in mind at the same time.

It may be helpful as a start in inspecting for a possible wave to mark off highs or lows by laying pencils upon them and shifting the pencils among the highs or lows until an apparent regularity is visible. The length of the apparent regular spacing of highs or lows as well as their positions give a basis for further study. A cycle of the indicated length, it should be emphasized, may or may not be found to be a reality. (Figure 2).

By holding a pencil between apparent highs or lows (Figure 3), and shifting the thumbed pencil back and forth across the graph of a series, an apparent length and rhythm may be found for later testing.

Graduated Scales.* A cycle believed to be present in a series on the basis of inspection or other indication may be tested further by a

*A GRADUATED SCALE MAY BE CONSTRUCTED FOR ANY LENGTH OF ASSUMED CYCLE BY LAYING A STRIP OF PAPER, AN INCH OR SO WIDE AND AS LONG AS THE GRAPH, AND MARKING OFF ON IT SHORT LINES AT INTERVALS EQUAL TO THE CYCLE LENGTH TO BE TESTED. ADDITIONAL GRADUATED SCALES MAY BE MADE IN A SIMILAR WAY FOR TESTING POSSIBLE RHYTHM OF OTHER LENGTHS IN THE CYCLE. SEVERAL GRADUATE SCALES OF EVEN LENGTH, SUCH AS FIVE, TEN, FIFTEEN, TWENTY, OR TWENTY-FIVE YEARS MAY BE USED IN RUNNING A GENERAL INSPECTION FOR APPARENT RHYTHM.

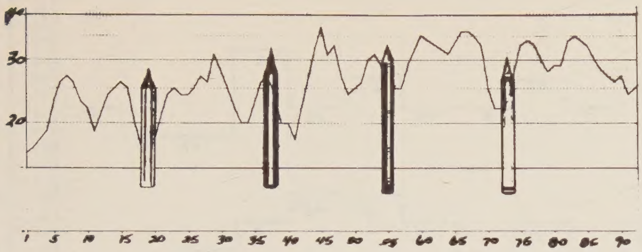


FIGURE 2. PENCILS MAY BE LAID ON APPARENT AREAS OF HIGH AND LOW TO AID THE EYE IN NOTING POSSIBLE RHYTHM AND WAVE LENGTH. THE SEVERAL PENCILS MARK OFF AREAS OF HIGH AND INDICATE A POSSIBLE 18-YEAR CYCLE. (THIS CURVE IS A CONSTRUCTED ONE HAVING A STRONG 18-YEAR CYCLE ALONG WITH 7.5, 8.0, AND 10.0-YEAR CYCLES).

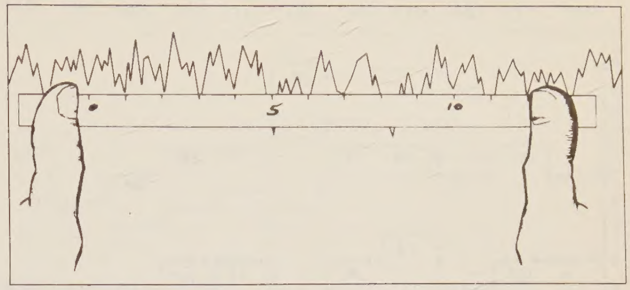


FIGURE 4. A SLIP OF PAPER MARKED OFF AT UNIFORM INTERVALS MAY INDICATE A RHYTHM AND A POSSIBLE CYCLE LENGTH. THIS 20-YEAR GRADUATED SCALE SHOWS AREAS OF HIGH THAT INDICATE A POSSIBLE 20-YEAR CYCLE.

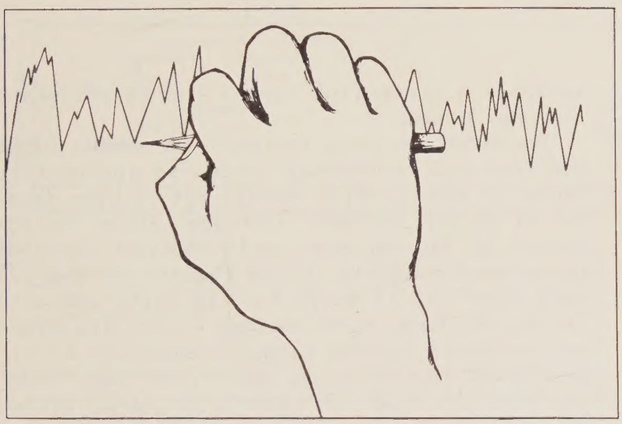


FIGURE 3. "THUMBING A PENCIL" TO APPARENT HIGHS OR LOWS MAY INDICATE THE POSSIBLE LOCATION OF A WAVE IN PLOTTED DATA.

graduated scale (Figure 4). In using a graduated scale, one should look for *areas of high* between *areas of low* and *vice versa*, rather than for single highs and lows for each point on the graduated scale. Any cycles of other lengths and any distortions may mask perfect repetition of a cycle, yet its strength may show through as areas of high or low, respectively. One should expect also that a high or low of a cycle could be obliterated by distortions; in addition, there may be extraneous highs and lows.

A comparison may make these points clear. The rhythmic pattern of city streets spaced one block apart may be distorted by the complete absence of a street or by an alley in the middle of one block. Yet the over-all rhythmic pattern of streets one block apart would remain the basic pattern.

By shifting the graduated scale slightly, one can make adjustments between the scale and the graph to indicate how much longer or shorter than the scale intervals the cycle may be. In this way, for example, a 20-year graduated scale may be used to reveal an 18-year cycle. In such a case, an 18-year graduated scale should then be used for an additional check before going further in the cycle analysis.

Observed highs and lows may mark turning points of cycles, as well as mark accidental events. They may also mark the blended effect of several cycles. Yet in some series, well marked highs and lows may be absent, though cycles may be present. A curve of rapidly changing trend may have no highs or lows, their place being taken by increased or decreased growth or decline of the trend itself. Because it is hardly likely that an increasing or decreasing trend will go on forever, the trend may be expected to flatten out and the normal appearance of highs and lows to become visible. Growth curves thus may show the influence of cycles as changes of the growth rate rather than as well-marked highs and lows.

Determining Length of Cycles. Several ways may be employed for measuring the length of a cycle, which fall into nine general categories that may be listed in order of increased precision (and complexity) as follows:

(1) Median or modal length of cycle intervals
(2) Average interval between terminal crests (*or* terminal troughs).
(3) Average interval between terminal crests *and* terminal troughs.
(4) Average interval between several respective terminal crests (or troughs).
(5) Graduated scales.
(6) Time charts.
(7) Comparison of data with fixed cycles of various wave lengths to find the best fit.
(8) Periodograms
(9) Periodic tables averaged by sections.

The intervals between the various successive highs or successive lows may be computed individually and (1) the average (median or modal) length determined as the length of the cycle. The method is very crude.

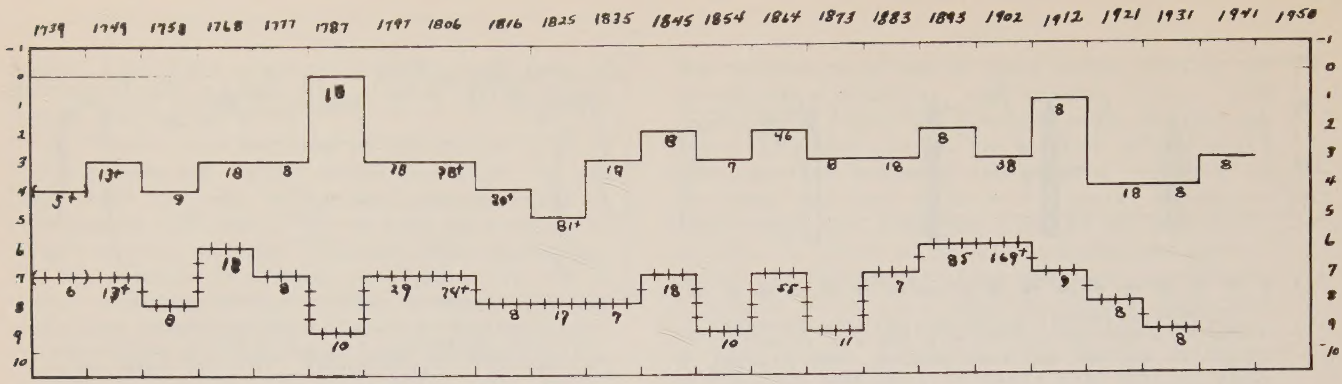


FIGURE 5. BANDS OF HIGH (UPPER) AND LOWS (LOWER) RUNNING HORIZONTALLY ACROSS A 9.6-YEAR TIME CHART INDICATE A CYCLE LENGTH OF 9.6-YEARS. THIS TIME CHART HAS BEEN PREPARED FROM CLEARSPAN NUMBERS OF THE LYNX DEVIATIONS OF LOGARITHMS FROM THE 9-YEAR MOVING AVERAGE. THE CLEARSPAN VALUE (C_sC_s) OF EACH HIGH AND LOW RESPECTIVELY ARE AS GIVEN. THE PARENTHESIS AROUND AN INITIAL HIGH OR LOW INDICATES ABSENCE OF INFORMATION FOR PREVIOUS YEARS, THOUGH THE CLEARSPAN NUMBERS THEMSELVES ARE OF STANDARD VALUE.

The average between terminal highs or lows (2) has already been described, as it and (3) are the easiest—though almost the least accurate—ways of measuring a cycle. But if by coincidence, an actual cycle has highs and lows precisely at the two observed terminal crests or troughs of the manifest cycle, the average length corresponds to that of the cycle itself. Averaging the interval between terminal crests and troughs will give a value that may also give an approximation of cycle length.

Intervals between several terminal highs or lows may be averaged (4) and the average position used in averaging the several intervals between the respective terminal positions thus established.

Graduated scales (5) may be applied to a curve to measure it for rhythm. A scale length that fits a series of highs and lows indicates a possible cycle length (Figure 4).

A time chart (6) shows a cycle of the same length as the chart in the form of horizontal bands of highs and lows (Figure 5). A cycle of a length differing from the time chart will have bands of highs and lows running diagonally across the chart (Figure 6). The slope of the band measures the length of the cycle as plus or minus that of the chart.

Cycles of various lengths may be applied to the intervals and the variations between the respective test cycles and the actual curve measured (7). The cycle length giving the minimum variation may be selected as representing the approximate length of a cycle in the data.

Periodograms (8) constructed by harmonic analysis indicate possible cycle lengths. By *multiple harmonic analysis,* the lengths of probable cycles may be indicated with greater precision.

Periodic tables may be averaged (9) by sections and the plus or minus slippage between sections applied to the length of the periodic table as a correction factor for establishing the true length.

The errors in using intervals to measure highs and lows and lengths of cycles is apparent in Tables 12 and 13 which consist not of Lynx data, but of controlled data. In these tables the intervals of the manifest cycle of eight combined known cycles (Table 12 and Figures 16 and 17) vary from 7 to 14 years for the highs and 7 to 11 for the lows, with medians of 10. The average intervals between terminal points at "1741" and "1944" for the highs and "1738" and "1948" for the lows give 9.67 and 9.55 years, rather close to and indicative of the alpha cycle of 9.6 years. We know the exact timing of the 9.6-year cycle, however, as well as of the other seven cycles (Table 12) because, being controlled data, they are present by construction. If the three terminal crests are averaged at each end of the series and the averages used as the new terminal highs, the average interval of the highs is 9.67; for the lows if similarly treated it would be 9.6.

Time Charts. One of the most of useful tools for cycle analysis is the *time chart*, and no serious cycle research should be done without freely using time charts. Because the time chart has not been described in general print, I shall describe it in more detail at this time.*

A time chart is a grid, on which one makes a graphic comparison of the successive highs with each other and successive lows with each other. In constructing a time chart (e.g. Figure 5) a series of uniform intervals are marked off hori-

*FOR A MORE COMPLETE DISCUSSION OF TIME CHARTS AND CLEARSPAN NUMBERS, SEE DEWEY, EDWARD R., 1949, *Cycle Analysis: A Description of the Hoskins Time Chart*. TECHNICAL BULLETIN NO. 3, FOUNDATION FOR THE STUDY OF CYCLES.

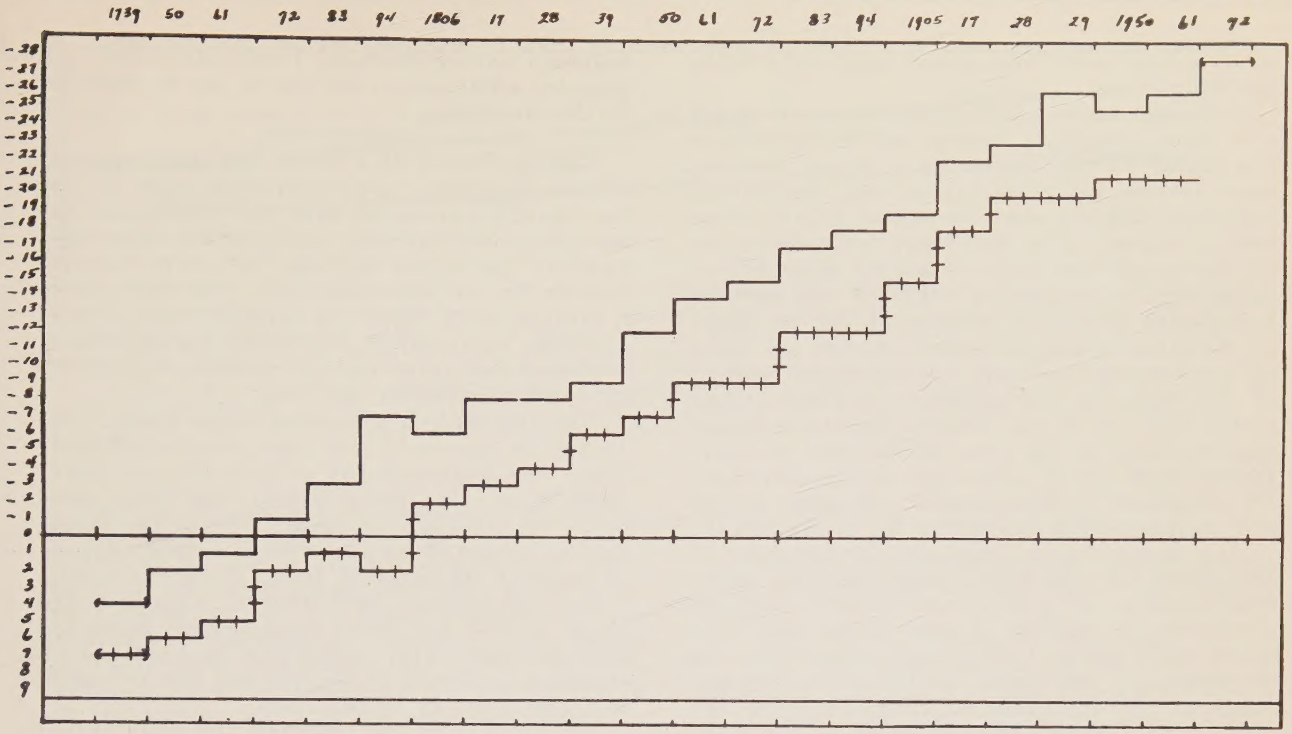


FIGURE 6. THE 11-YEAR TIME CHART OF THE DATA IN FIGURE 5 STILL SHOWS A 9.6-YEAR CYCLE. THE RISE (TOWARDS THE UPPER RIGHT) INDICATES A CYCLE SHORTER THAN THE INTERVALS OF THE TIME CHART RELATIVE TO THE SLOPE OF THE LINE — IN THIS CASE 1 1/2 YEARS LESS THAN 11.1 OR 9.6-YEARS.

zontally near the top of a sheet of graph paper, the points approximately an inch apart. (For making a time chart, I suggest a cross section paper ruled five lines to the inch). At the left margin one then rules off a vertical scale as long as the cycle to be tested. The time chart begins with the first year of record, not according to any highs or lows.

Figure 5, referred to previously, shows a 9.6 year time chart of deviations of logarithms of Lynx abundance from their 9-year moving average. As will be explained, this has been prepared from the clearspan numbers of Table 2, but it can be prepared directly from the curve of Figure 1. In the 9.6-year time chart, the highs form a horizontal band centered at the third year after the base years, which indicates the average timing of the highs. There are always some variations from a true straight line as in the third cycle. Such variations may be accounted for (a) by the fact that the time chart (and data) is in calendar years and the cycle in 9.6 years, (b) by random events influencing the Lynx index, and (c) by the influence of other cycles.

A straight line is then fitted to the highs and another straight line to the lows. If these straight lines are horizontal, the cycle has a length equal to the length of the cycle length of the grid.

Time charts often indicate cycles having lengths differing from that of the base intervals. In Figure 6, a time chart of 11.1 years shows highs and lows to rise to the upper right. The rise of a straight line fitted to the high is from position-2 after base 1750 to position-28 before base 1972 (the high of 1944 will plot as base 1972 minus 28), a rise of thirty positions through twenty intervals. This indicates a cycle length of 9.6 years. (A time chart interval of 11.1 years less 30/20 or 1 1/2 years). The run of lows show a rise of thirty-one positions in twenty-one intervals, which indicates a length of 9.62 years (11.1 − 31 ÷ 21).

Clearspan Numbers. The highs and lows entered upon a time chart may be determined subjectively by examining a graph, but a much better and objective method is to determine them through the use of clearspan numbers. Clearspan numbers may be prepared from the original figures, from their moving averages, or from any deviations from moving averages. They may be prepared from whole numbers, logarithms, or percentages. In case of a series having a rapidly changing trend, however, as in a growth curve, the clearspan determined from original figures may lack significance. In such a case, deviations from trend or moving difference or ratios should be employed. Clearspan numbers provide

an objective means for determining cycle highs and lows.

Clearspan numbers are of two kinds, *black* and *red*. Black clearspan numbers may be defined as the number of time units (e.g. years, months, days) between any point higher than the preceding value and the next preceding point of the same or higher value. Red clearspan numbers are the number of time units between a point of low value and the preceding point of the same or lower value. Clearspan numbers of (a) are those of the highs, clearspan numbers of (b) are those of the lows. In practice, the clearspan numbers of the highs (a) are tabulated in black, those of the lows (b) in red. Table 2 shows the clearspan numbers of the Lynx deviations of logarithms from their nine-year moving average. The clearspan numbers usually tabulated in red are indicated in the table by an asterisk.

In a sense, black clearspan, for example, is the number of time units backwards to a datum over which a value will not "slide", and red clearspan the number of time units back to a datum under which it will not "slide." That is, the number of time units "in the clear." We may think of the deviation of 2.334 in 1896, for example, as sliding *under* the value of 2.470 for 1895 and colliding with the *lower* value of 2.255 for 1894. Therefore, its "clearspan" covers only the year 1895 and is 1 *in red*. But the clearspan of the 1899 value of 1.409 is intercepted only by the lower value of 1.396 in 1813. Therefore its clearspan number is 85. The clearspan number may be determined by counting the free spaces or by subtracting the year following the intercepting year from the year whose clearspan is being determined. For 1889, this would be obtained by subtracting 1814 from 1899 to obtain a red clearspan number of 85. The clearspan for the 1813 value of 1.396 passes beyond the 1739 beginning of the data, a fact that is indicated as a red 74^{+}

Black clearspan numbers for a datum higher than the preceding datum are obtained in the same way except that one now determines the interval *over* which the value will slide to an earlier intercepting value. The "clearspan" of the 1895 value of 2.470 is intercepted by the higher value of 2.496 in 1886 to give a clearspan number of 8 (1895 — 1887). Black clearspans passing beyond the start of the data are indicated with a + as in the red clearspans (*e. g.* 81^{+} for 1820).

Because the alpha cycle often tends to overshadow all others and to mask any cycle web present, it may be necessary to subtract it from the data before any further time-chart reconnaissance may be carried out. Table 3 shows the clearspan numbers of the deviations of the data from the 9-year moving average after the 9.6-year cycle, 9.0-cycle, and 7.46-year cycle have been removed and the results smoothed by a weighted moving average. From this table, for example, additional time charts may be made for further analysis.

Turning Points of a Cycle. The determination of turning points (points of cycle high or low) has been a source of much confusion. As has been mentioned earlier, a subjective determination of the highs or lows may give varying lengths for any apparent cycle. For this reason a turning point should be selected objectively. *A turning point may be defined as a time unit of standard clearspan next preceding a standard clearspan of opposite amplitude.*

The standard of clearspan value usually selected is one-half the time chart interval. Thus, the standard for a 20-year time chart would be 10, that for a 10-year time chart would be 5. For intervals of uneven length, the standard is selected as the whole number nearest to one-half the interval (e.g., 5 for 9.6 years, 4 for 7.8 years.) *Substandard clearspan* are those between one-third the standard value and the standard value. They may be used when a standard clearspan is missing and are indicated on a time chart by broken lines.

The symbol for the standard clearspan selected is capital C followed by the standard value as a subscript. Small c is used to indicate substandard clearspan. Thus the symbol for standard clearspans of 8 and substandard clearspans of 3 would be C_8c_3.

Reversing Cycles. In any web of cycles, inter-cycle interference or merger may be present. Just how much this complicates measurement and examination depends upon the relationship of the cycles. Long cycles usually influence short cycles least, but the closer the cycles approach each other in lengths, the greater the possible interference. The closer the lengths, the longer will the series need be to separate or "unscramble" the cycles.

Two cycles may blend together so as to give an apparent cycle of intermediate length. The intermediate will persist for a number of repetitions (the number depending upon the degree of closeness of length of the two generating cycles). Eventually the intermediate or "false" cycle will decline in amplitude, after which the cycle reappears *but in opposite phase* or "upside down" in relation to a projection of the false cycle. The highs thus come when a person depending upon the false length would have expected lows and vice versa. Such a cycle is termed a *reversing cycle* and the apparent length is termed the *false length*. The two cycles composing a reversing cycle blend together at the least common multiple of their length. The interval between two such points of reversal is termed the *synodic period*. It is the key to the length of the two component cycles.

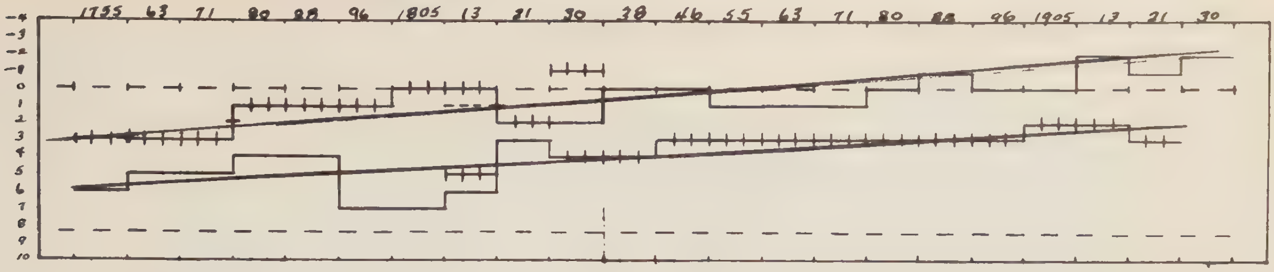


FIGURE 7. A TIME CHART OF 8.33-YEARS LENGTH SHOWS A REVERSING CYCLE WITH A FALSE LENGTH OF 8.14 YEARS. THE BAND OF LOWS RUNS INTO THE BAND OF HIGHS AT THE 1830 INTERVAL WHERE THE BAND OF HIGHS RUNS INTO THE LOWS. THE SLOPE OF THE REVERSING BANDS INDICATES A CYCLE SHORTER THAN THE TIME-CHART INTERVALS. THE TIME CHART HAS BEEN MADE FROM THE CLEARSPANS OF TABLE 3. IN *Cycles, A Monthly Report* FOR JANUARY 1952, VOLUME 3, PAGE 8, E. R. DEWEY CALLED ATTENTION TO THE PROBABLE PRESENCE OF WHAT SEEMED TO BE A COMPOUND WAVE OF SLIGHTLY MORE THAN EIGHT YEARS IN LENGTH. THIS COMPOUND WAVE IS EVIDENTLY A REVERSING CYCLE.

When the synodic period is known, the lengths of the two component cycles may be found. The first step is to determine the number of times the cycle of false length has been repeated in the synodic period. The synodic period is then doubled and the doubled length (a) divided by one cycle more than double the number of false cycles to give the length of the shorter cycle and (b) divided again by one interval fewer than twice the number of false intervals to give the length of the longer cycle. Thus, a reversing cycle with a false length of 10 years and with a synodic period of 165 years (16 1/2 false intervals long) would be brought about by two cycles of 9.71 and 10.31 years length. The computation for determining this follows:

$$\frac{165 \times 2}{32} = 10.31 \qquad \frac{165 \times 2}{34} = 9.71$$

Several such reversing cycles appear, though none has been found that reveals its full synodic period. A time chart using 8 1/3 year interval indicates a cycle with a reversing length of 8.14 years (Figure 7). The bands of lows and highs respectively run into each other at the 1830 interval and each continues the respective course of the other. The reversal point is in or near the 1830 interval. We can fix one reversal point more or less accurately, but the next one is beyond the graph to the right. Because there are at least twelve intervals between the reversal point and the end of the graph, it is obvious that the next reversal point must be more than twelve cycles from that of 1830. (It could be the next interval off the graph). This indicates that the synodic period is at least 12 cycles long or 97.7 years (12 x 8.14). To find the limits of the two merging cycles, we may proceed as follows:

False length of reversing cycle: 8.14 years

Synodic period: at least 12 intervals or at least 97.7 years (12 x 8.14)

Double number of false intervals to date: 24

Minimum synodic period divided by double the number plus one: $\frac{195.4}{25} = 7.82$

Minimum synodic period divided by double the number minus one: $\frac{195.4}{23} = 8.49$

True lengths of generating cycles indicated (if this behavior represents a true reversing cycle):

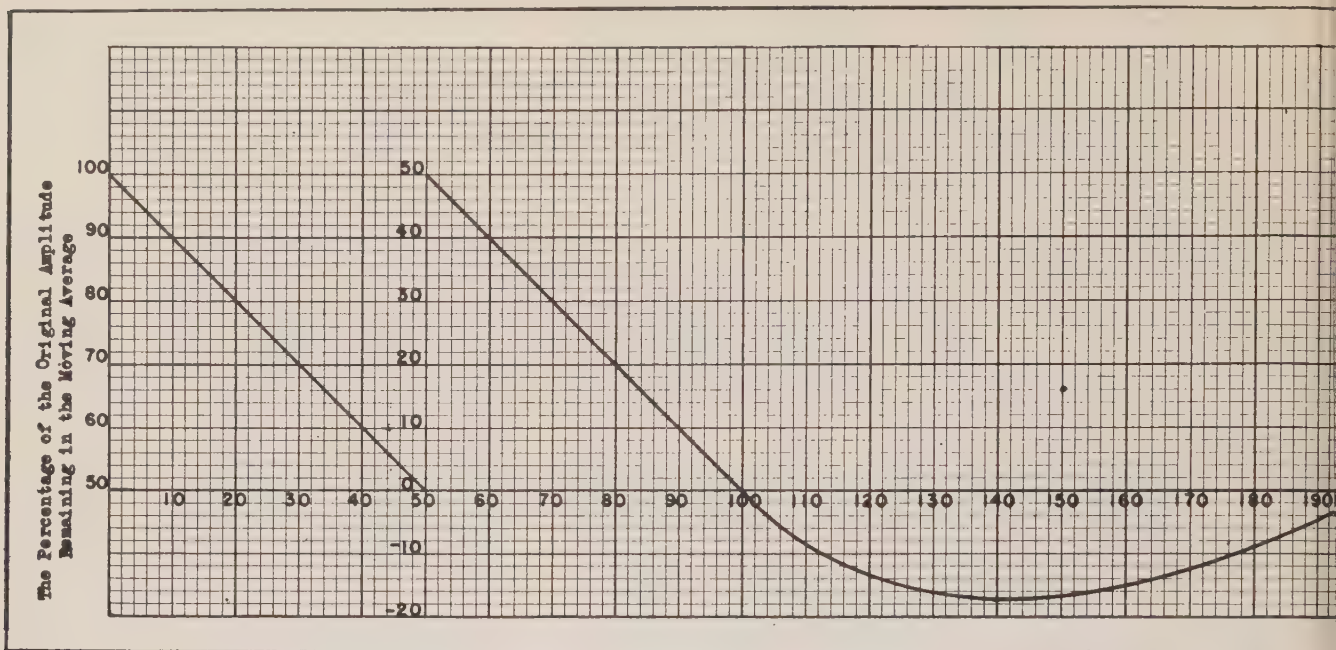
(a) 7.82 years or shorter and
(b) 8.49 years or shorter

In addition to the 8.14 year reversing cycle, the 8 1/3 year time chart indicates also a cycle of 8.43 to 8.63 years (average 8.53) or perhaps two cycles of these respective lengths. An 8-year time chart shows an 8-year length in the band of highs and a 7.82 year length in the lows. Because we cannot determine the exact synodic period until another reversal takes place, we do not know whether or not cycles indicated on the time chart as 7.82 and 8.43 are the ones producing the 8.14 reversing cycle. Further analysis, however, may determine this if the two cycles can be separated.

Moving Averages. A moving average is one of the most powerful tools used in manipulating data to emphasize, minimize, or reveal cycles. As this use of the moving average may not be completely understood by biologists and others, a brief discussion may clarify the manipulations of this study.*

A moving average of the same length as a cycle completely eliminates that cycle and all cycles of submultiple or harmonic length (e.g. 1/2, 1/3, 1/4). It will nearly eliminate cycles of nearly submultiple length (almost harmonic). Thus, a 27-year moving average eliminates cycles of 27, 13.5, 9.0, 6.75, 5.4, etc. years.. In addition, it eliminates various parts of other cycles, such as about, twenty-seven thirtieths or 90% of a 30-year cycle. *But a moving average introduces no cycles, and it does not change the length of any cycle in any series of figures.*

*FOR A MORE COMPLETE DISCUSSION OF THE MOVING AVERAGE IN CYCLE ANALYSIS, SEE DEWEY, EDWARD R. 1950, *Cycle Analysis: The Moving Average.* TECHNICAL BULLETIN NO. 4, FOUNDATION FOR THE STUDY OF CYCLES.



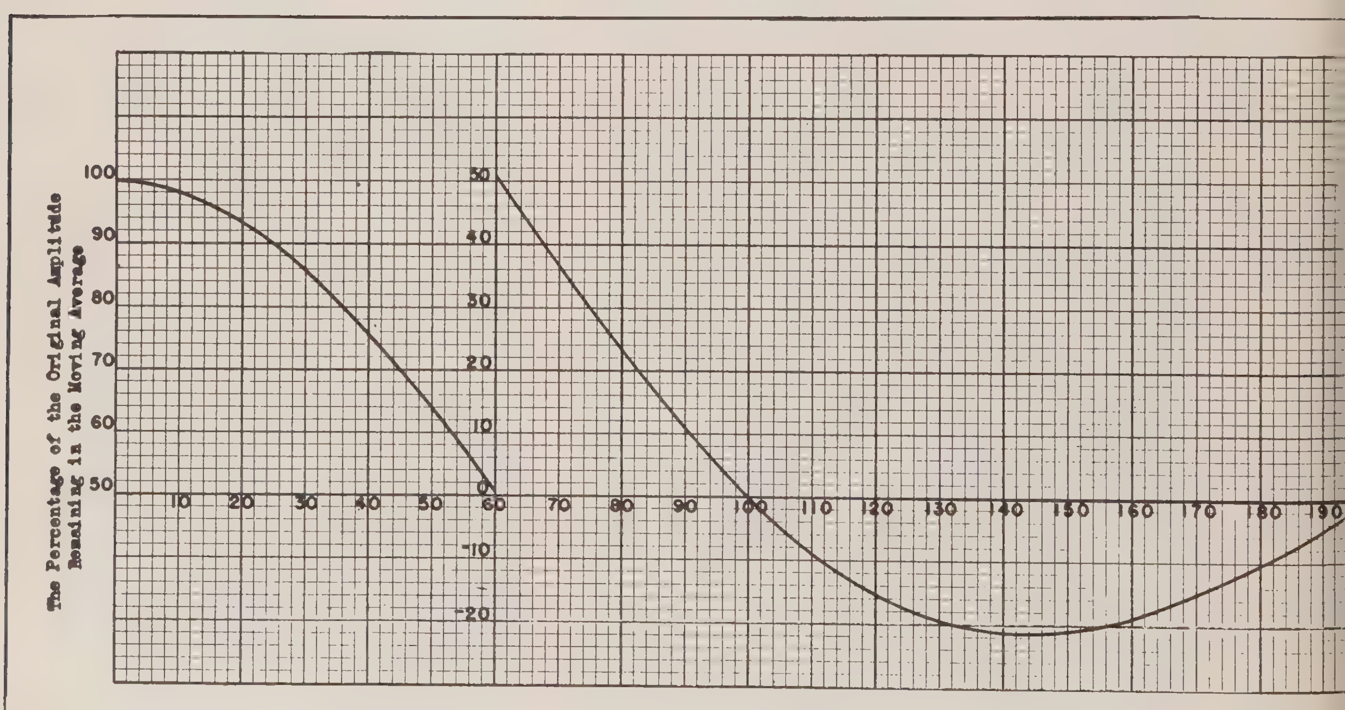
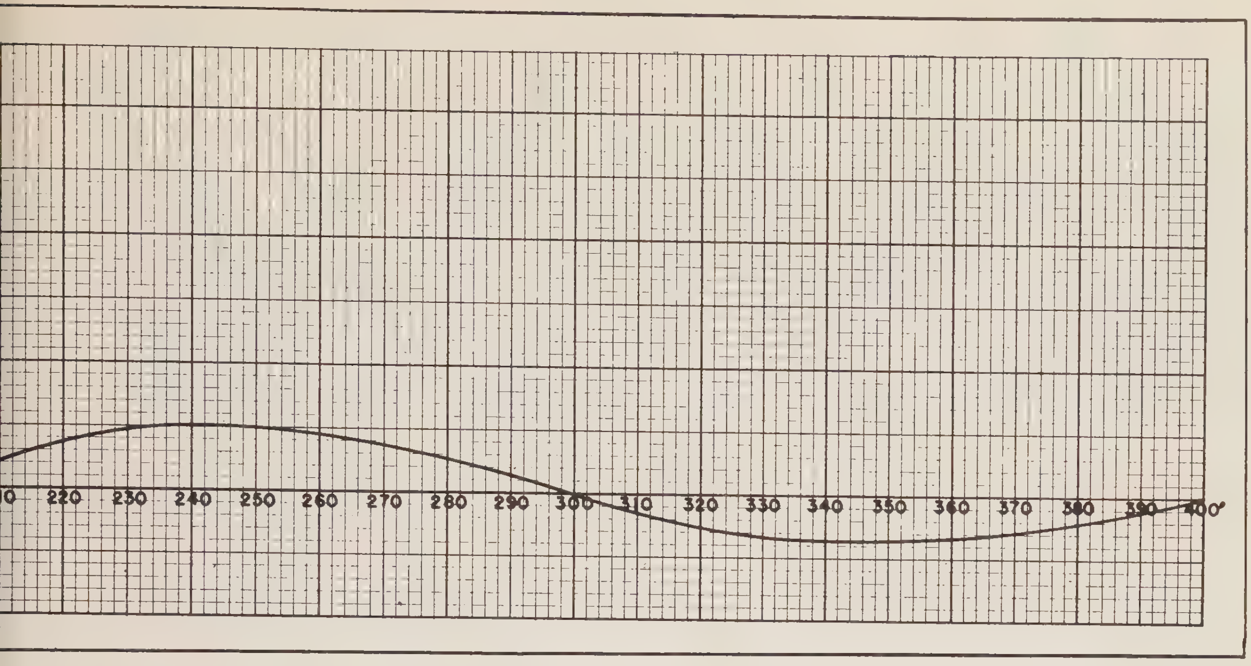
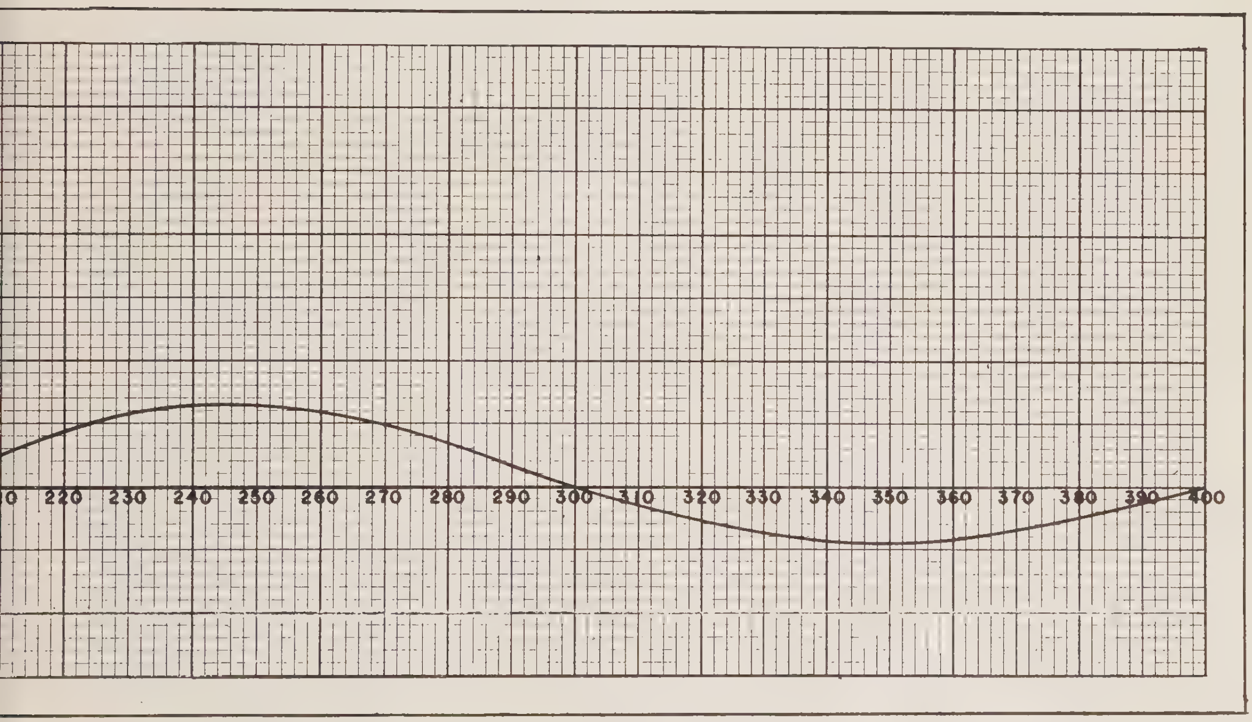


FIGURE 8 FOOTE CHART FOR DETERMINING AMPLITUDE RETAINED IN A MOVING AVERAGE FOR A ZIG-ZAG CURVE (TOP) OR A SINE CURVE (BOTTOM). A 9-YEAR MOVING AVERAGE (50% THE LENGTH OF AN 18-YEAR CYCLE) WOULD RETAIN ABOUT 64% OF THE AMPLITUDE OF AN 18-YEAR CYCLE. IT WOULD RETAIN





100% OF THE AMPLITUDE OF A 9-YEAR CYCLE (HENCE, ELIMINATE IT). THE 9-YEAR MOVING AVERAGE (150% THE LENGTH OF A 6-YEAR CYCLE) WOULD RETAIN 21% OF A 6-YEAR CYCLE, AND THE 6-YEAR CYCLE WOULD REAPPEAR IN THE 9-YEAR MOVING AVERAGE AND IN REVERSE BY 21% AMPLITUDE. (FROM: DEWEY, EDWARD R., 1950, IBID.)

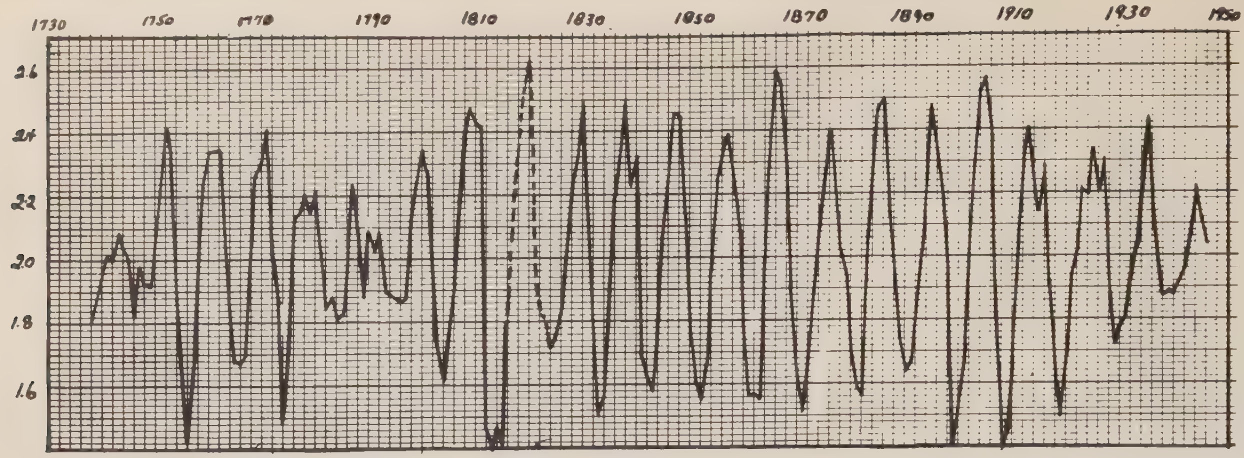


FIGURE 9. DEVIATIONS OF LOGARITHMS OF THE LYNX FROM 9-YEAR MOVING AVERAGE. THE MOVING AVERAGE IS PLOTTED AS THE BASE (LOGARITHMS 2.000 OR 100%). WHICH HAS THE EFFECT OF CONVERTING THE DASHED LINE OF FIGURE 1 INTO THE STRAIGHT LINE SHOWN HERE. THE YEARS INFLUENCED BY INCREASING THE INDEX AT 1820 (SEE TEXT) ARE SHOWN AS BROKEN LINES.

Its influence is upon the *amplitude,* which it may decrease (to zero and hence elimination for cycles of its own length) or increase. A moving average longer than a cycle may *reverse* the cycle, except for harmonics which it eliminates. A 27-year moving average, for example, eliminates any 27- and 13.5-year cycles, but it reverses any between 13.5 and 27 years long. Cycles between 9.0 and 13.5 years would be in phase. But cycles between 6.75 and 9.0 would be reversed again and so on.

Restoring the Amplitude of a Cycle. Because a moving average may decrease or increase the amplitude of a cycle not of its own length, it may be necessary to correct the amplitude at some point in the analysis. Otherwise one must select a moving average of the cycle length for each cycle being studied. Because of the complications involved in centering even-length moving averages or in making decimal moving averages (such as 9.6 for the 9.6 year cycle), it is most convenient to use either the nearest odd length moving average or a moving average that will nearly eliminate the greatest number of interfering cycles. In the Lynx data, the 9-year moving average is the nearest odd length to the suspected alpha cycle (9.6 years). The 9-year moving average retains about 6.5% of the amplitude of any 9.6-year cycle which can be adjusted at any time by a correction factor proportioned to the retained 6.5%. The means for restoring the amplitude lost to any moving average makes it possible to test many cycles and yet compute but one moving average. The labor involved is thus markedly reduced.

The amount of the original amplitude of each cycle remaining in the moving average varies with the length of the moving average and the shape of the curve. Figure 8 presents the Foote Chart for determining the percentage of the original amplitude retained by the moving average. By reference to Figure 8, it will be seen that the 9-year moving average retains for a sine curve 64% of the amplitude of an 18-year cycle but—21% of a 6-year cycle. The latter means that the cycle is reversed. In actual practice, the amplitude is restored to the deviations of the logarithms from the moving average. The deviations of the 9.6-year cycle from the 9-year moving average would each be increased by a 6.95% (93.5:6.5) to correct for the moving average. Those for any 18.0-year would each be increased by 56.2% (64:36) to correct for the influence of the 9-year moving average upon any 18.0-year cycle. But the deviations of any 6-year cycle from the 9-year moving average would be decreased by 26.6% (79:21) because the amount retained was 21%.

When the deviations from a moving average are graphed, the cycles eliminated by the moving average emerge. This is shown in Figure 9 where the 9.6-year cycle and others appear again. The effect of plotting the deviations is to use the moving average as a base from which the deviations are plotted. It is as though the moving average were pulled and pushed into a straight line, which in logarithms has the value of 2.000 (or 100%).

Periodic Tables. Having located apparent cycles by inspection, graduated scales, or time charts, the validity of the cycle, its length, amplitude, shape, and phasing may be tested by periodic tables of the length of the presumed cycle. Table 4 is the periodic table with which the 9.6-year cycle has been measured. If this one comes "on time", its appearance in the second half of the periodic table will match that of the first half. Dividing the periodic table in sections and comparing the curve in the two sections tests the length of the cycle. If the cycle is longer than the table, the curve of the second section will slip to the *right* with reference to the first. If shorter, the slippage will be to the *left*.

Figure 10 shows the two halves of the 9.6-year periodic table of the deviations of the logarithms from the 9-year moving average. The slippage of the second half from the position of the first half (Base 1805 to Base 1872) is less than a tenth year in the seven cycles. The length may be fixed with considerable precision as that of the periodic table (9.6-years). A slippage of one-tenth of a year in seven cycles would be one-seventieth of a year in one cycle, which would indicate the cycle length as one-seventieth of a year longer or shorter than the length of the table itself. It will be seen thus that the precision of measurement varies with the number of cycles in the table. A slippage of one-tenth of a year would fix the cycle length within .014 years but the same slippage in the longer record with fifty repetitions of the 9.6-year cycle would fix the cycle length within .004 years (one-tenth slippage in twenty-five cycles). In the case of the 9.6-year cycle, the slippage is clearly less than a tenth of a year in seven cycles, which indicates that the true length will not be shorter than 9.59 years nor longer than 9.61 years.

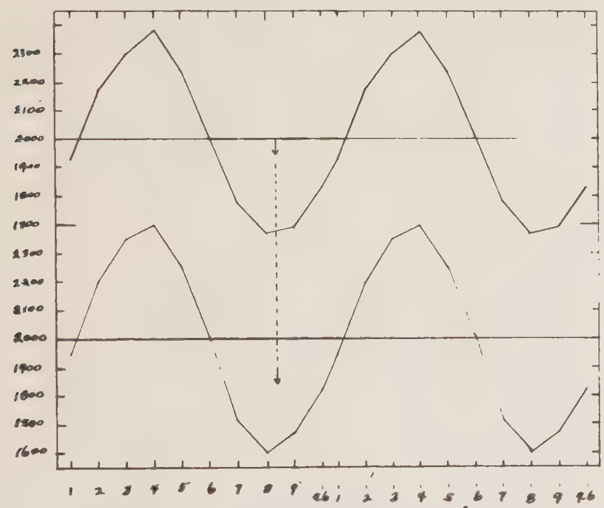


FIGURE 10. THE RESPECTIVE AVERAGES OF THE TWO SECTIONS OF TABLE 4 SHOW NO SLIPPAGE AND THEREFORE INDICATE A CYCLE LENGTH THE SAME AS THAT OF THE TABLE OR 9.6-YEARS.

Overlapping Sections of a Periodic Table. A cycle other than that being tested may interfere with the results obtained by a periodic table. Thus, a cycle one year longer than a 9.6-year cycle would distort the results of averaging a table by halves except when the halves coincide in length with the cycle length. But if the table is divided into overlapping parts, each of as many intervals as the length of the interfering cycle, the latter will appear in each column proportionately and thereby average itself out.

This can be illustrated more clearly by simple numbers than by the Lynx computations, but the practice is the same. Table 5 shows a 5-year periodic table having both a regular 5-year cycle of simple numbers (1,2,3,3,2,) and a 6-year one (1,2,3,4,3,2,). The numbers totalling more than about average are entered in a work table in black, those totalling less than about average are entered in red. The purpose of colors in the work sheets is to suggest visually any strong cycles slipping across the columns of the table, to the *right* if longer than the table and to the *left* if shorter than the table. The 6-year cycle slipping across the five-year periodical table (Table 5) is indicated by a line slanting downward to the right. The 5-year cycle slipping across a 6-year periodic (Table 6) shows a slanting line to the left. The averaging of Table 5 by halves (A) gives a representation of both cycles, regular parts of the 5-year, varying parts of the 6-year. Actually, the five-year cycle is distributed regularly in all columns, so that the totals by halves neither distort nor reveal it.

The 6-year cycle, however, slips one year in each five-year cycle(five years in five cycles). so that a five-section average carries only 5/6 of it. But a six-section average (0 - 25 or 20 - 45) will have a complete 6-year cycle distributed equally in all columns of the six sections. Hence, it will be averaged out as is shown under B, where subtracting the 6-year average of 2.5 reveals the 5-year cycle of 1, 2, 3, 3, 2, which was placed therein.

In Table 6, the same figures have been placed in a 6-year periodic table of twelve lines. The 5-year cycle does not average out when the table is sectioned by halves. But it does when 10-line overlapping sections are used (B). Subtracting the average 5-year cycle (2.2) from the section averages reveals the 6-year cycle (1,2,3,4 3,2) as also placed therein. This manipulation illustrates the important point that a multiple of a cycle length may be used for sectioning a periodic table and will eliminate the cycle as completely as the cycle length itself.

The periodic tables for the Lynx deviations (in logarithms) have been handled substantially as in this example of simple whole numbers.

Average Median. The average median of the section of a periodic table usually proves more representative than the arithmetic average. It is obtained by checking off successively the highest and lowest valves until the number predetermined as the number to use for the average median has been reached. In the 9.6-year periodic table (Table 4), for example, the seven median values were used. In a 7.9 periodic table with 17-cycle overlapping sections (to eliminate the influence of an 8.4-year cycle), eleven median values were used (Table 7). In the 26-cycle table used in measuring the amplitude of the 7.95-year cycle, fourteen median values were used in computing the average median.

Slippage as a Measure of Cycle Length. The section averages are graphed and 5-to-the-inch

cross-section paper as used in making time charts is recommended for such use. In graphing the average of a periodic table, the data are plotted through two whole runs so that a complete cycle will appear regardless of the position of highs and lows in the columns of the periodic table. For this reason, the base year for both the first and second runs on the graph is the beginning of the record irrespective of the high or low.

In Figure 11, for example, the crest falls at approximately position 7 (Column 7) of Table 7. By carrying through a full two runs, a complete representation of the cycle from high to high and low to low is assured. Because the 7.9 position (Column 7.9) will not fit exactly the line of the graph, it is plotted as position-8. Position-1 of the second run is thereupon plotted as 1.9 years from position-7. In this way, the difference between the calendar year of the graph and the decimal year of the cycle is adjusted.

The nadir point of the lows of the respective curves of the two halves of the periodic table is determined by locating the crossing points of the axis (the 2.00 or 100% line using deviations in logarithms) and marking off the midpoint between the crossings. In Figure 10, the crossing points indicate that the midpoints of the lows of both sections are nearly half-way between eight and nine lines. Because they fall exactly in the same vertical position, no slippage is indicated and establishes the length as 9.6 years. But in Figure 11, the midpoint of the lower curve (second section of the periodic table) measures 7/16 of a position to the right of the midpoint of the upper curve. There are nine cycles between the centers of the two overlapping sections. This slippage of the lower curve *to the right* of the upper indicates that the true length of the cycle is *longer* than 7.9 years by a ninth of the slippage (1/9 of 7/16) of .05 years. The cycle measures 7.95 years.

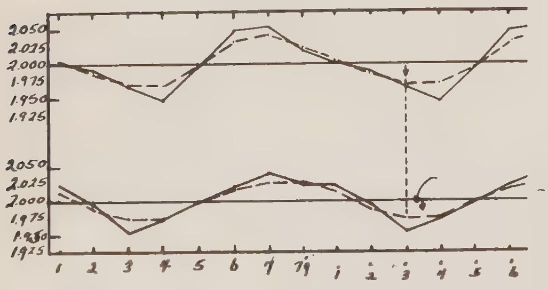


FIGURE 11. SECTION AVERAGE MEDIANS OF A 7.9-YEAR PERIODIC TABLE (TABLE 7) SHOW SLIPPAGE TO THE RIGHT OF ABOUT 7/16 OF A YEAR IN 9 CYCLES. THIS INDICATES A CYCLE LENGTH LONGER THAN THE TABLE BY A NINTH OF THE SLIPPAGE OR .05-YEARS. THE 3-YEAR MOVING AVERAGES OF THE AVERAGE MEDIANS ARE SHOWN AS BROKEN LINES. THE CENTER OF THE TWO LOWS IS FIXED AS MIDWAY BETWEEN CROSSING POINTS OF THE AXIS (2.000).

The presence of other cycles or random distortions, may make it difficult to locate the midpoints because of the unevenness of the two curves. A moving average of the average medians may reveal the truer path of the curve. In Figure 13, a 3-position moving average of the average median is shown as a broken line.

A slippage of the lower curve *to the left* would indicate a cycle length *shorter* than the periodic table by an amount equal to the total slippage divided by the number of cycles between the centers of the two sections, just as the slippage to the right indicates a proportionately longer cycle.

Determining Cycle Amplitude. When deviations from the moving average trend are being used in making a periodic table, the average median (or other average of the columns) indicates the amplitude of the cycle. But the amplitude indicated varies from the true amplitude by amounts equal to that retained in the moving average. Restoring the amplitude by use of the Foote Chart (Figure 8) has already been described.

When the deviations being used are those of one moving average from another moving average, a more complicated correction is involved. Table 8 is a 15-year periodic table of the deviations of the 11-year moving average of the Lynx logarithms (after removal of the 9.6 year cycle) from the 23-year moving average. Eleven-years was used because

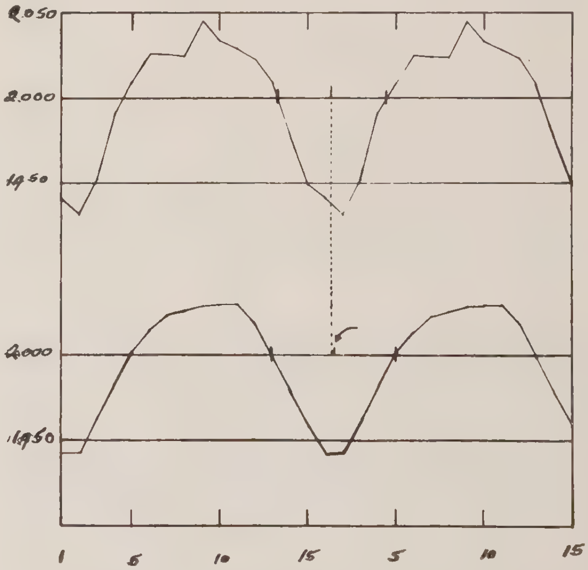


FIGURE 12. A SECOND SECTION AVERAGE MEDIAN OF TABLE 8 SLIPS TO THE RIGHT ABOUT A QUARTER OF A YEAR IN THE 4.87 YEARS BETWEEN THE CENTER POSITION OF THE TWO SECTIONS. THIS INDICATES A CYCLE LENGTH .05 YEARS LONGER THAN THE 15-YEARS OF THE TABLE OR 15.05 YEARS.

of an apparent cycle near 12 years in length and 23 years because of a cycle indicated as longer than 22 years. Figure 12 shows that the second half of the table (Base 1872) slips to the right a quarter of a position from the first half of the table (Base 1799). Because there are some 4.87 cycles between Base 1872 and Base 1799, the slippage measures .05 years per cycle, which indicates a true length .05 years longer than the 15 years of the table or 15.05 years.

The 11-year moving average has a length of 73.3% of the 15.05 year cycle while the 23-year moving average has a length 152.8% of it. By reference to the Foote Chart (Figure 8) it can be determined that for a sine curve the 11-year moving average retained 32% of the original amplitude of the 15.05-year cycle; at the same time the 23-year moving average retained-21%. We must therefore, correct the amplitude of the 15.05-year cycle by a positive 32% and a negative 21%. This may be done in the following manner:

x = deviations of original value from 11-year moving average.

y = deviations of 11-year moving average from 23-year moving average.

z = deviations of original value from 23-year moving average.

100% W = original value.

$$x + y + z = 100\%\ W.$$
$$y + z = 32\%\ W.$$
$$z = 21\%\ W.$$
$$x = 100\% - 32\%\ W$$
$$x = 68\%\ W$$
$$xy = 100\%\ W - (21\%\ W)$$
$$xy = 121\%\ W$$
$$y = 121\%\ W \quad x$$
$$y = 121\%\ W - 68\%\ W$$
$$y = 53\%\ W$$
$$W = \frac{y}{53\%}$$
$$W = \frac{100\ y}{53}$$
$$W = 1.89\ y$$

Hence, to restore the original value of the amplitude, each separate value of the deviations of the 11-year moving average from the 23-year moving average (y) is multiplied by the restoration factor 1.89. In calculating this, the amount of the deviation from the 2.000 axis (100%) is increased by 1.89 and added back to the axis. Thus, the value of 1.952 for the first column is .048 *below* the 2.000 axis. When increased by the factor of 1.89, it becomes .081 below the axis, which added back (negatively) to the axis of 2.000 gives a new value of 1.909. The value of 2.029 at position-9 is .029 *above* the 2.000 axis. When this is increased by 1.89 it becomes .055 above the axis (positively) to give a new value of 2.055. In a like manner, the value for each datum of the original amplitude may be restored. The new data will be logarithms of the pure cycle of 15.05 years.

Fitting a Curve to the Cycle Values. Two serviceable methods of fitting curves may be used in cycle analysis, (a) mathematical and (b) graphical. Though the steps involved are laborious, fitting a curve by mathematical equations has little really practical advantage over graphical fitting. For most cycle studies, graphical curve fitting usually will be found suitable enough.

The shape of the curve determines the type to be used; zig-zag (rectilinear) or sine-shaped As has been stated earlier, observed biological data generally show sine curves. (We do not, know however, if this is so for the cycle generants) Economic and other non-biological data usually show zig-zag patterns. Biological data tend to show symmetrical curves, which may also be a general rule.

Because few cycles in a time series, if any (other than seasonal cycles) show regular calendar lengths, some adjustment of cycle length and calendar length must be made. This may be done by fitting a curve to as many cycle intervals as make an even number of calendar years. In the case of the 9.6-year cycle, the least number of cycles would be five to make forty-eight calendar years.

Because of the short length of the Lynx record, (216 years) a slight variation from a true multiple introduces no significant error. A curve for the 15.05-year cycle would have to be fitted to twenty intervals (301 years) for perfection. But one fitted to a 15-year interval will be but .365 years off at the beginning of the record, correctly phased at the middle and but .365 years off at the end, 7.3 cycles from the middle. There are but 6.1 repetitions of the 35.2-year cycle, and a 35-year fitting will be out of phase but .6 of a year at the beginning and end of the record. In the same way, the curve for the 7.46-year cycle was fitted on the basis of 13-intervals, which total 96.98 years. The error of .02 years is not significant in the 216-year series. For the reasons of practicality, therefore greater precision or perfection ordinarily need not be sought.

Figure 13 shows procedure for fitting a five-interval, 9.6-year curve to the average median of Table 4 and Figure 10. The fitting grid is set up on graph paper ruled five lines to the inch. Each fifth vertical line marks the position of each point in the cycle and each fifth horizontal line marks .100 of the logarithms. Because there are five intervals of 9.6 years each, there are 48 vertical lines and each one will then represent a year. The value for each year may be read off the graph from the crossing points of the curve with respect to the vertical lines (years) and the horizontal lines (deviations in logarithms). The methods of numbering each of the forty-eight years is as shown.

Phase Determination. The phasing of a cycle (placing of highs and lows in the series), may be determined readily from the fitted curve and periodic table. In the fitted curve of the 9.6

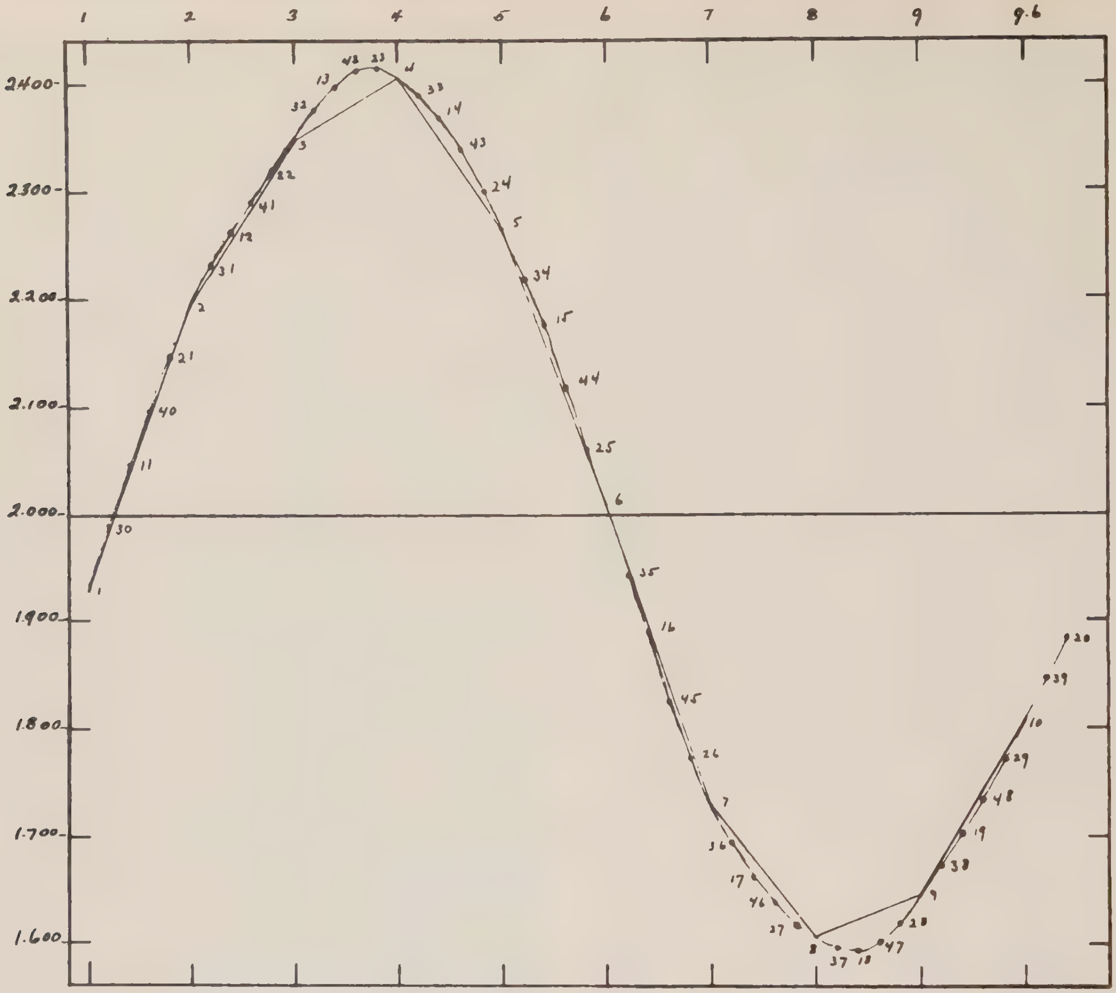


FIGURE 13. A SINE CURVE MAY BE FITTED TO THE DATA OF 9.6-YEAR PERIODIC TABLE BY FIVE INTERVALS (48 YEARS) TO ADJUST THE CYCLE LENGTH TO THE CALENDAR. EACH OF THE FORTY-EIGHT YEARS WILL HAVE A POSITION ON THE GRAPH AS SHOWN, AND THE LOGARITHMS OF THE 9.6-YEAR CYCLE FOR EACH YEAR MAY BE READ OFF THE GRAPH AS NUMBERED. (GRAPH PAPER RULED IN LIGHT BLUE LINES SPACED FIVE TO THE INCH WAS USED).

year cycle (Figure 13) the first position of the curve is that of the first column in the periodic table (Table 4). Because the first column of the table bears the year 1739 (one year after the base year of 1738), the first position of the fitted curve is also the year 1739. The fitted curve thus adjusts the 9.6-year cycle to the calendar for forty-eight years (until 1789), then repeats for forty-eight years (until 1835); it repeats again beginning at 1883 and 1931. Periodic tables started with different base years would still indicate the initial year of the fitted curve as the first year of the respective tables. Hence, the phasing of a cycle is wholly objective.

Inter-cycle Influences. Many events and circumstances limit the efficiency of cycle analysis. Facetious though it may sound, it has been said with considerable truth that only after a series of data has been subjected to a complete analysis is one ready to determine the cycles, their amplitude, and their timing, Obviously, the presence of one or more cycles in a series will influence the analysis, unless one can eliminate known cycles by appropriate manipulations of the data. Manipulations that eliminate or neutralize inter-cycle influences often may be use of simple or weighted moving averages, moving differences, moving ratios, and adjustments to isolate a cycle.

Removing a Cycle from the Data. Although influence of a cycle may be neutralized by appropriate moving averages and by sectioning a periodic table. One cycle, (such as the alpha cycle of the Lynx) may mask the influence of others, or its length may be too near that of another for aver-

aging out. It may be desired at any time, therefore, to "lift" a cycle from the data so that its influence may be eliminated.

Obviously, how completely one has established the exact amplitude and shape of the cycle will determine how completely it will be removed. If the representation of the cycle as subtracted is not as great as the amplitude of the actual cycle, some of the cycle will be left in the data. But if too great, the data will be over-corrected and a reverse cycle of the same wave-length will be introduced. The correction for a cycle may be checked by setting up a new time chart and periodic table of the cycle length to determine if any residual or overcorrection exists in the newly corrected data.

A cycle may be removed by subtracting its values from the appropriate data. This is illustrated in Table 9 in which the data of Table 6 have the 6-year cycle (as measured by the 10-line overlapping section) removed to reveal the five-year cycle. The subtraction of the 5-year cycle as measured by the 6-line overlapping section averages of Table 5 reveals the 6-year cycle. No correction of amplitude is needed for these section averages, because they do not represent deviations from a moving average (*e.g.* Table 4).

In removing a cycle by subtracting deviations in logarithms, one subtracts the difference between the respective deviation and the axis (2.000). This has the mathematical effect of taking the deviations out by percentage. A cycle is removed from a series of deviations containing the cycle or from any logarithms of numbers that contain the cycle. Thus, the cycle may be removed from the original curve as well as from deviations from a moving average of that curve.

Table 1 includes the removal of the 9.6 Lynx cycle from the original curve, which gives a new curve without the masking influence of the 9.6-year cycle (Figure 14). The 9.6-year cycle may be removed from the deviations of the logarithms from the 9-year moving average. Because the 9-year moving average was found to have retained 6.5% of the 9.6-year wave, however, use of the restored 6.9 wave would remove too much amplitude from the deviations,(though the correct amount from the original logarithms). Hence, one would have to use the value of the 9.6-year cycle as shown by the deviations or reduce the restored value by 6.5% before removing. In practice, it may be easiest to subtract deviations from the periodic table and to enter new values in a table of the same construction. Figure 15 shows a graph of the deviations after the 9.6-year cycle has been removed; it should be compared with Figure 10.

After removing or neutralizing a cycle, such as the 9.6, one is ready to repeat the steps used previously. By inspection, graduated scales, time charts, additional cycles may be indicated for further measurement with periodic tables. Other cycles found to be true may be removed from the data successively if desired as was done for the 9.6-year cycle.

Synthesizing a Curve of Cycles. A curve may be constructed from corrected values of several cycles by adding the difference between the axis and the deviations (just opposite of the practice used in removing a cycle). Obviously, the phase of the several cycles must be as in the

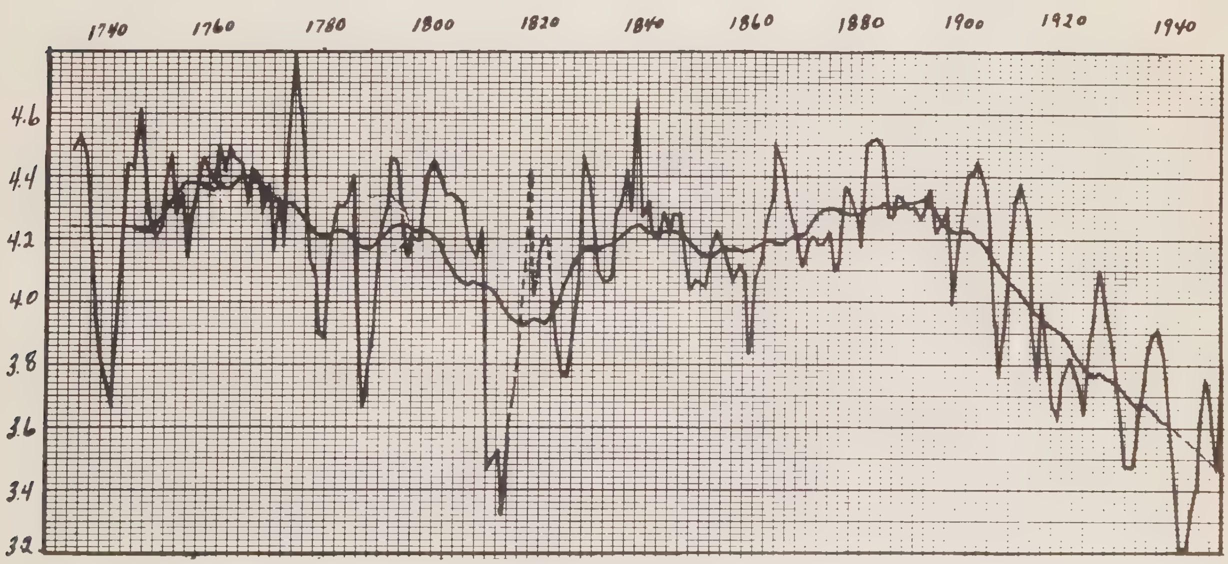


FIGURE 14. THE 9.6-YEAR CYCLE HAS BEEN REMOVED FROM THE LOGARITHMS OF THE LYNX INDEX. THIS GIVES A NEW CURVE WITHOUT THE MASKING INFLUENCE OF THE 9.6-YEAR CYCLE. A 23-YEAR MOVING AVERAGE OF THE NEW CURVE IS SHOWN AS A HEAVY LINE. THE BROKEN LINE INDICATES THE ZONE INFLUENCED BY SPLICING THE INDEX.

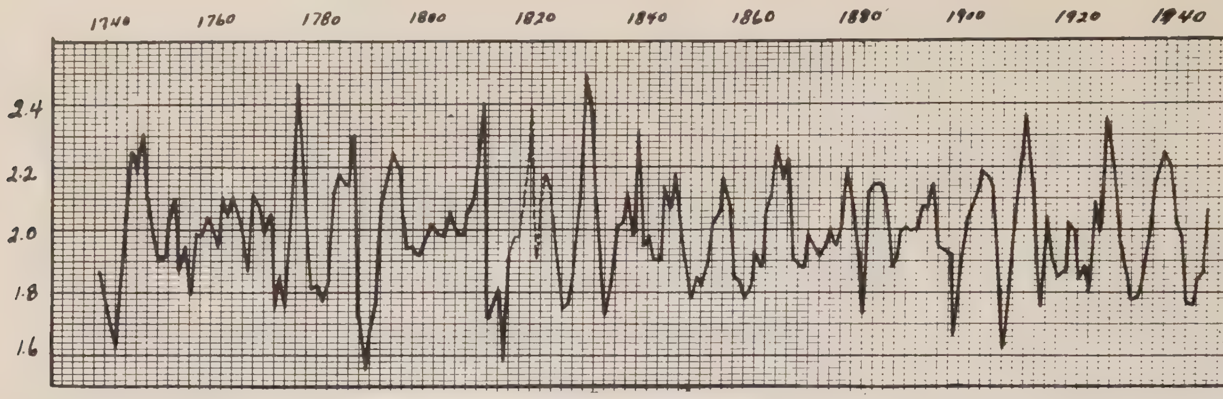


FIGURE 15. THE 9.6-YEAR CYCLE HAS BEEN SUBTRACTED FROM THE DEVIATIONS FROM THE 9-YEAR MOVING AVERAGE (FIGURE 11). THIS GIVES A NEW SET OF DEVIATIONS FROM THE 9-YEAR MOVING AVERAGE WITHOUT THE MASKING INFLUENCE OF THE 9.6-YEAR CYCLE. THE BROKEN LINE SHOWS THE ZONE OF SPLICING OF THE INDEX.

original curve—*i.e.*, the highs and lows in their correct calendar position (Table 4).

The synthesized curve will be the sum of the several cycles and represent the cycle elements as measured in the original curve. It will vary in appearance from the original curve with (a) the precision of measurement, (b) any undetected and hence unincluded cycles, (c) the number of noncyclic changes in the series, and (d) the amount of the trend. The eight cycles isolated in the Lynx data (9.6, 15.05, 11.75, 9.0, 7.95, 22.2, 7.46, 35.2) have varying amplitude and phases as shown in Table 10, and Figure 16.

EIGHT LYNX CYCLES

7.46-year Cycle. Time charts and apparent slippage in periodic tables indicated a possible *cycle web* in the general six to nine year range. Only three of them have been isolated, the shortest of them measuring 7.46-years length, with an amplitude of 113.8% of trend at high and 87.9% of trend at the low. Measurements with a 7.5-year periodic table and its graphical representation from two sections of twenty cycles each (average median of fourteen items) show slippage of 1/3 years in eight cycles (Base 1806 to Base 1866) or .04 years per cycle Hence, the true length is fixed mathematically at 7.46 years. A 7.43-year periodic table shows a slippage of 1/4 years in eight cycles or 1/32 of a year for each. This confirms the true length of the cycle as 7.46 when carried to two decimals.

7.95-year Cycle. A cycle indicated in periodic tables and time charts to be between a tenth to a twentieth of a year less than 8.0 years in length has been established by a 7.95-year periodic table with 17-cycle overlapping sections and 8-item average median as 7.95 years long. A graph of the averages shows a positive slippage of 7/16 years in the nine cycles represented between the middle of the two sections (Base 1801 to Base 1872). This shows the slippage to be about .05 years per cycle and the length of the cycle as 7.95 years. Measurements by a 7.95-year periodic table show an amplitude of 122.5% of trend at the high and 81.7% at the low.

9.0-year Cycle. Because a length apparently a little less than 9.0 years has been indicated by time charts and graduated scales, a periodic table of 8.9-years length was set up for the data after removal of the 9.6-year cycle. The positive slippage of about 3/4 of a year during seven cycles indicates a cycle of about a tenth year longer than the length of the table or 9.0 years. The average amplitude measures 126.5% of trend at high and 79.1% of trend at low.

9.6-year Cycle. The 9.6 cycle already has been mentioned. In the course of the analysis, it was subtracted from the index to give a residual in which the seven other cycles listed were measured (Figure 9).

11.75-year Cycle. Indications of several cycles in the general neighborhood of eleven to twelve years appear in several time charts as well as in graphs of the data when pointed by means of graduated scales. But of this group, only a cycle of 11.75-years length has been isolated. By means of an 11 7/8-year periodic table, the length has been determined to be 11.75-years. It has an amplitude of 134.9% of trend at the high and 74.1% at the low.

15.05-year Cycle. A "down beat" of some forty-five years length appears in some runs of

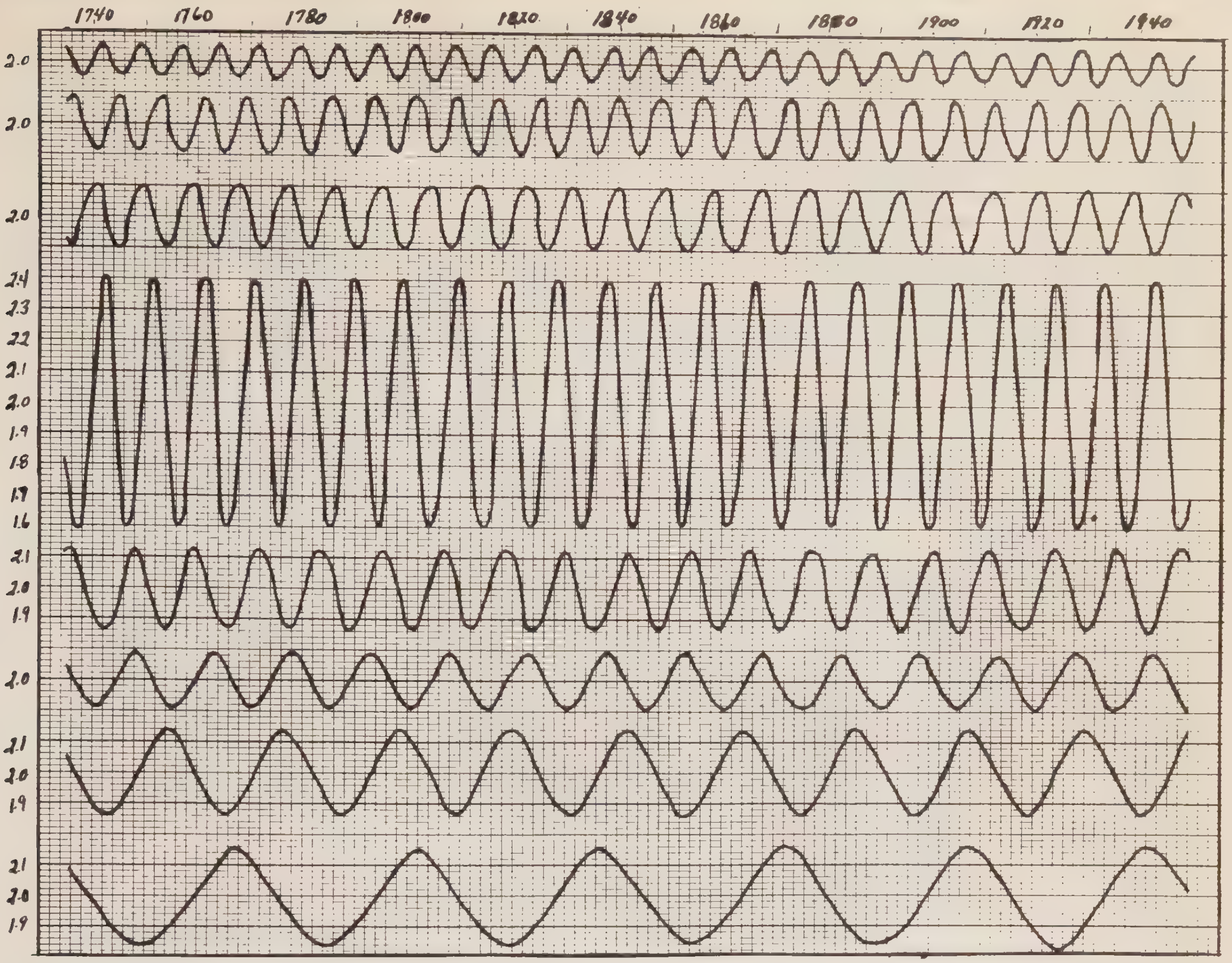


FIGURE 16. THE EIGHT PURE CYCLES ISOLATED FROM THE LYNX FUR RETURNS ARE SHOWN GRAPHICALLY. THE AMPLITUDES ARE INDICATED AS PLOTTED.

lows after removal of 9.6-year cycle. It suggests that two or more cycles may have a multiple near that length. Because the first two submultiples are 22 1/2 and fifteen years (half and third of forty-five, respectively), cycles of near these lengths were tested. A 15-year periodic table (Table 8) showed that the length was slightly more by about a twentieth of a year (Figure 12). The cycle of 15.05-years has an amplitude of 122.5% of trend at the high and 81.7% of trend at the low.

22.2-year Cycle. A cycle of possible length one-half that of the down-beat proves rather difficult to isolate. The presence of a web of cycles in the twenty to twenty-three year range appears evident. The components of this web have not been established, but a possible reversing cycle of 21.3-years (composed of one 19.76-years or shorter and one of 22.4-years or shorter) and another reversing cycle of about 23.8-years (probable components not tested) indicate the complex nature of the web. A cycle with a length of 22.2-years is shown by a 22.2-year periodic table and graph. It is also indicated by 25-and 22.2-year time charts. The amplitude of 138.0% of trend at the high and 72.4% of trend at the low indicates considerable strength and suggests the possibility that it is the Lynx gamma cycle.

35.2-year Cycle. A time chart reconnaissance suggested a cycle in the 32-to 35-year range. By means of 32-year, 34-year, and 35-year periodic tables and appropriate graphs, the length is established as 35.2-years for six repetitions in the 216-year data. The amplitude measures 144.5% of trend at the highs and 69.2% at the lows, which indicates that the 35.2-year cycle is the Lynx Beta cycle. The variations of time charts and periodic tables in the 32-to 35-year range, however, suggests the possible presence of some additional cycles.

Timing of the Lynx Cycles. The various years of highs and lows of a regular curve may be determined from a periodic table. The probable

calendar years of highs and lows for pure cycles of the eight lengths measured are given in Table 10. The probable amplitude at the highs and lows is given for the several cycles in Table 11.

Because we have a measure of the timing and amplitude of the eight curves, it is possible to represent each one graphically, as in Figure 16, from the data for the eight cycles, (Table 12).

Combined Effects of Cycles. The eight cycles reinforce or offset each other according to their respective parts in contributing to the whole. If all cycles were known and measured correctly, their combination when added to the noncyclic part of the record would give the original curve except for the trend. The amplitudes of the eight cycles (Figure 16) have been added together (positively and negatively as the case may be) as logarithms, which has the effect of building curve according to the percentage of the amplitudes (themselves percentages of trend). These have been graphed in Figure 17 from the combined total of Table 17. Figure 17 is thus the *manifest* cycle of the eight periodicities derived from the Lynx index.

The manifest cycle of the combined eight cycles shows distinctly the power of the alpha cycle. The timing of the manifest cycle varies from a true 9.6 years by the necessities of the terrestrial calendar year and by the influence of the other seven cycles. As we know the composition of the manifest cycle (See Table 12), it should not be surprising that difficulties arise in trying to fix by inspection the length of unknown cycles in raw data. It would seem obvious that serious cycle study requires more refined analytical methods than mere inspection and guessing. Table 13 illustrates this further by showing the highs and lows of the manifest cycle of the eight derivations. The highs vary from seven to fourteen years apart and the lows from seven to eleven years. They average about 9.6.

Forecasting from Cycle Deviations. Because the residual (Figure 18) contains cyclic elements as yet unmeasured, the combined eight cycles as a basis for forecasting suffers from our not knowing these. Obviously, the residual should be analysed. Furthermore, we have no way of accounting for noncyclic events, though we can estimate the possible course of the trend. To make a forecast from the cyclic elements, it is necessary to project the *trend*. This would be done in Figure 1 by continuing the apparent direction of the trend line. Logarithms of this projected trend for each year may then be read off the graph. The pure cycles may be combined and their logarithms for future time continued (as for 1951-1954 in Table 12) and added to the corresponding logarithms of the projected trend. The sums will represent the respective forecasts in logarithms and may be converted to whole numbers by using a table of logarithms. Figure 19 projects the eight.

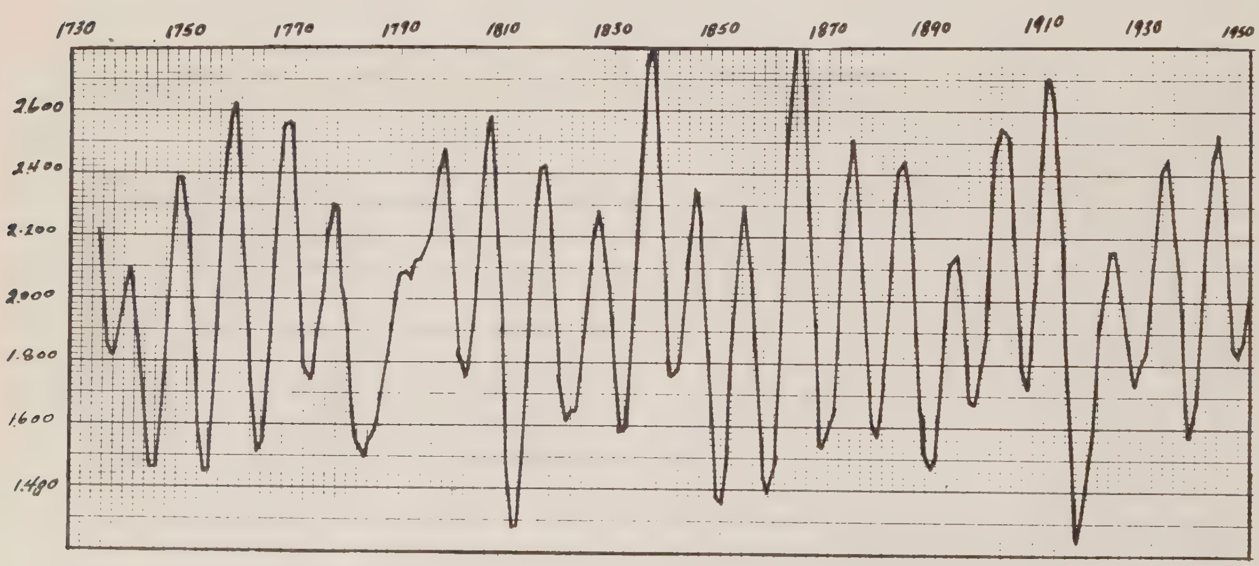


FIGURE 17. THE EIGHT PURE CYCLES ISOLATED IN THE LYNX INDEX HAVE BEEN COMBINED INTO A MANIFEST CYCLE. IT WILL BE NOTED THAT THE INTERVALS VARY IN LENGTH, EVEN THOUGH DOMINATED BY AN ALPHA CYCLE OF 9.6 YEARS. ONLY EIGHT OF THE MANY PROBABLE LYNX CYCLES HAVE BEEN USED, BUT THE COMBINED EFFECT GIVES A REPRESENTATION SOMEWHAT SIMILIAR TO THE LYNX INDEX OF FIGURE 1. A COMBINED CYCLE LIKE THIS MAY BE USED AS THE BASIS OF A FORECAST BY ADDING IT TO ("WRAPPING IT AROUND") A PROJECTION OF THE TREND.

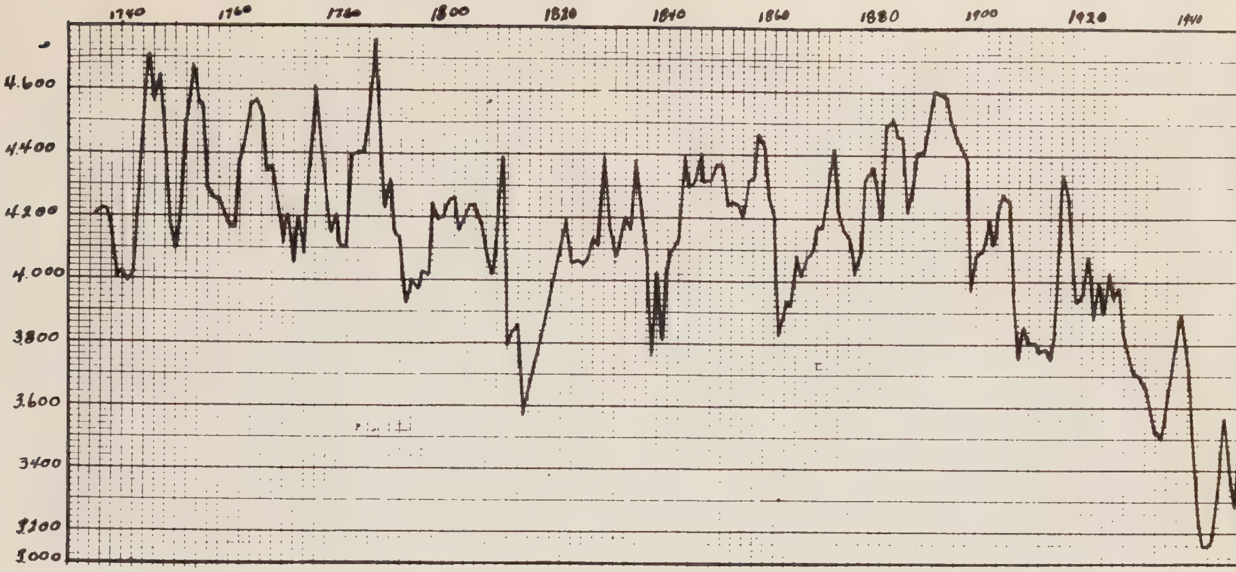


FIGURE 18. THE RESIDUAL OF THE LYNX INDEX (FIGURE 1) AFTER THE EIGHT CYCLES, (FIGURE 16 AND 17) HAVE BEEN REMOVED. THE PRESENCE OF ADDITIONAL AND UNANALYZED CYCLES IS EVIDENT.

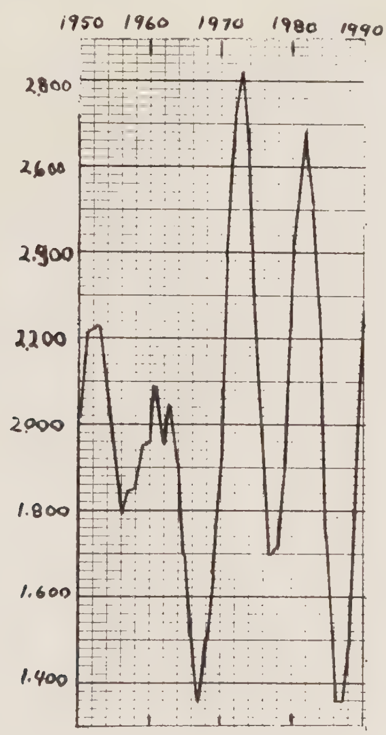


FIGURE 19. A MANIFEST OF THE EIGHT CYCLES WILL COMBINE AS SHOWN IF THEY CONTINUE THE PATTERN OF THE PAST. THIS CURVE CAN BE CONVERTED TO A PROBABLE PERCENT OF TREND AS AN EXTENSION OF FIGURE 17. IT SHOULD BE NOTED THAT TO BECOME A BASIS OF A FORECAST, THE TREND AND UNANALYZED COMPONENTS OF FIGURE 18 SHOULD BE TAKEN INTO ACCOUNT. IN ADDITION, RANDOM EVENTS WILL DOUBTLESS INFLUENCE THE YEARLY FIGURE. IN ACTUAL PRACTICE NOT ONLY SHOULD THE RESIDUAL BE ANALYZED BUT THE DERIVED CYCLES OF THIS MANIFEST SHOULD BE RE-EXAMINED AS EACH NEW YEARLY FIGURE BECOMES AVAILABLE.

TABLE I

Lynx Indexes, Moving Averages and Deviations from Moving Averages

Year	Index	Logarithms	9-Year Moving Average	Deviations from 9-Year Moving Average	Ideal 9.6-Year Cycle	Logs of Residual	11-Year Moving Average of Residual	23-Year Moving Average of Residual	Deviations of 11-Year from 23-Year Moving Average
1735	20,593	4.314			1.825	4.489			
1736	15,297	4.187			1.638	4.549			
1737	11,685	4.068			1.600	4.468			
1738	9,455	3.976			1.730	4.246			
1739	7,783	3.891	4.088	1.803	1.930	3.961			
1740	10,412	4.018	4.072	1.945	2.196	3.822	4.133	4.242*	1.891*
1741	12,242	4.088	4.070	2.018	2.348	3.740	4.128	4.242*	1.886*
1742	11,769	4.071	4.066	2.005	2.404	3.667	4.133	4.242*	1.891*
1743	15,187	4.182	4.097	2.085	2.262	3.920	4.132	4.242*	1.890*
1744	14,829	4.171	4.139	2.032	2.008	4.163	4.134	4.242*	1.892*
1745	14,714	4.167	4.171	1.997	1.728	4.440	4.157	4.242*	1.915*
1746	10,749	4.031	4.214	1.817	1.604	4.427	4.193	4.242	1.951
1747	17,869	4.252	4.276	1.976	1.643	4.609	4.252	4.238	1.969
1748	18,510	4.267	4.339	1.928	1.810	4.457	4.325	4.234	2.091
1749	20,477	4.311	4.392	1.919	2.042	4.269	4.358	4.232	2.126
1750	29,932	4.476	4.392	2.084	2.268	4.208	4.374	4.237	2.137
1751	42,174	4.625	4.389	2.236	2.377	4.228	4.347	4.261	2.086
1752	56,362	4.751	4.333	2.418	2.366	4.385	4.335	4.287	2.048
1753	44,141	4.645	4.301	2.343	2.176	4.469	4.314	4.320	1.994
1754	14,829	4.171	4.296	1.875	1.888	4.283	4.314	4.354	1.960
1755	10,054	4.002	4.310	1.692	1.662	4.340	4.328	4.377	1.951
1756	5,574	3.746	4.324	1.422	1.595	4.151	4.343	4.383	1.960
1757	9,655	3.985	4.328	1.657	1.700	4.285	4.368	4.343	1.985
1758	18,979	4.262	4.345	1.917	1.885	4.377	4.371	4.382	1.989
1759	40,407	4.607	4.374	2.227	2.145	4.461	4.374	4.368	2.006
1760	55,699	4.746	4.409	2.336	2.325	4.421	4.390	4.364	2.026
1761	61,210	4.787	4.446	2.341	2.410	4.377	4.399	4.359	2.040
1762	63,282	4.801	4.454	2.347	2.300	4.501	4.413	4.364	2.049
1763	30,416	4.483	4.434	2.050	2.062	4.421	4.427	4.362	2.065
1764	18,500	4.267	4.413	1.854	1.773	4.494	4.429	4.366	2.063
1765	11,969	4.078	4.401	1.678	1.620	4.458	4.413	4.380	2.033
1766	11,422	4.058	4.386	1.671	1.671	4.441	4.408	4.395	2.013
1767	11,874	4.075	4.382	1.693	1.766	4.309	4.389	4.397	1.992
1768	26,367	4.421	4.371	2.050	1.988	4.433	4.372	4.397	1.975
1769	43,163	4.635	4.370	2.265	2.232	4.403	4.350	4.388	1.962
1770	45,529	4.658	4.347	2.312	2.375	4.283	4.350	4.367	1.983
1771	57,540	4.760	4.351	2.410	2.390	4.370	4.380	4.343	2.037
1772	24,179	4.383	4.393	1.991	2.219	4.164	4.396	4.333	2.063
1773	18,163	4.259	4.397	1.862	1.942	4.317	4.404	4.330	2.074
1774	7,436	3.871	4.381	1.490	1.690	4.181	4.377	4.321	2.056
1775	12,274	4.089	4.356	1.732	1.598	4.491	4.348	4.317	2.031
1776	28,765	4.459	4.327	2.132	1.670	4.789	4.313	4.313	2.000
1777	23,901	4.461	4.311	2.150	1.845	4.616	4.270	4.294	1.976
1778	30,626	4.486	4.283	2.203	2.092	4.394	4.273	4.260	2.013
1779	27,545	4.440	4.300	2.140	2.300	4.140	4.272	4.239	2.033
1780	31,026	4.492	4.284	2.208	2.410	4.082	4.283	4.216	2.067
1781	17,374	4.240	4.222	2.018	2.338	3.902	4.268	4.207	2.061
1782	10,307	4.013	4.177	1.836	2.118	3.695	4.233	4.207	2.026
1783	10,444	4.019	4.160	1.859	1.825	4.194	4.178	4.211	1.967
1784	8,771	3.943	4.135	1.808	1.638	4.305	4.112	4.223	1.889
1785	7,972	3.902	4.081	1.820	1.600	4.302	4.083	4.224	1.859
1786	11,453	4.059	4.020	1.977	1.730	4.329	4.067	4.222	1.845
1787	21,560	4.334	4.097	2.237	1.930	4.404	4.094	4.210	1.884
1738	16,302	4.212	4.118	2.094	2.196	4.016	4.130	4.185	1.945
1789	10,202	4.009	4.126	1.882	2.348	3.661	4.154	4.171	1.983
1790	16,859	4.227	4.144	2.083	2.404	3.823	4.166	4.172	1.994
1791	14,724	4.168	4.147	2.021	2.262	3.906	4.169	4.186	1.983
1792	16,260	4.211	4.125	2.086	2.008	4.203	4.152	4.299	1.953
1793	10,402	4.017	4.123	1.894	1.728	4.289	4.135	4.218	1.917
1794	11,485	4.060	4.175	1.885	1.604	4.456	4.151	4.238	1.913
1795	12,200	4.087	4.216	1.870	1.643	4.443	4.210	4.244	1.966
1796	13,746	4.138	4.272	1.866	1.810	4.328	4.264	4.245	2.010
1797	15,471	4.190	4.314	1.876	2.042	4.148	4.313	4.243	2.070
1798	30,269	4.481	4.350	2.131	2.268	4.213	4.331	4.236	2.095
1799	39,250	4.594	4.350	2.244	2.397	4.197	4.336	4.225	2.111
1800	47,275	4.675	4.333	2.342	2.366	4.309	4.326	4.234	2.092
1801	38,472	4.585	4.323	2.262	2.176	4.409	4.316	4.226	2.090
1802	21,907	4.341	4.326	2.014	1.888	4.453	4.316	4.211	2.105
1803	11,474	4.050	4.325	1.735	1.662	4.398	4.326	4.195	2.131
1904	8,624	3.936	4.324	1.612	1.595	4.341	4.322	4.156	2.166
1805	11,064	4.044	4.314	1.730	1.700	4.344	4.317	4.125	2.192
1806	16,701	4.223	4.297	1.926	1.885	4.338	4.310	4.093	2.217
1807	29,322	4.467	4.292	2.175	2.145	4.322	4.225	4.066	2.159
1808	38,419	4.585	4.202	2.381	2.325	4.259	4.137	4.056	2.081
1809	38,419	4.585	4.119	2.475	2.410	4.174	4.057	4.068	1.989
1810	27,271	4.436	4.009	2.426	2.300	4.136	3.964	4.059	1.905
1811	19,814	4.297	3.883	2.414	2.062	4.235	3.894	4.059	1.836
1812	1,756	3.245	3.782	1.462	1.773	3.471	3.838	4.054	1.784
1813	1,283	3.108	3.712	1.396	1.620	3.488	3.794	4.038	1.756
1814	1,378	3.137	3.679	1.460	1.617	3.522	3.779	4.012	1.761
1815	1,220	3.086	3.675	1.411	1.766	3.320	3.801	3.985	1.816
1816	3,649	3.562	3.713	1.849	1.788	3.574	3.790	3.960	1.830
1817	8,887	3.949	3.792	2.157	2.232	3.717	3.783	3.940	1.843
1818	16,123	4.208	3.875	2.332	2.375	3.833	3.851	3.928	1.923
1819	30,510	4.484	3.950	2.535	2.390	3.094	3.901	3.934	1.967
1820	43,415	4.638	4.018	2.629	2.219	4.419	3.932	3.940	1.992
1821	8,896	3.954	4.035	1.918	1.942	4.012	3.973	3.942	2.031
1822	7,173	3.856	4.026	1.830	1.690	4.166	3.990	3.940	2.050
1823	6,456	3.810	4.009	1.809	1.598	4.212	4.007	3.933	2.074
1824	5,104	3.708	3.990	1.718	1.670	4.038	4.027	3.959	2.068
1825	5,161	3.713	3.963	1.750	1.845	3.868	4.061	3.993	2.068
1826	7,254	3.861	4.033	1.828	2.092	3.769	4.058	4.027	2.031
1827	11,550	4.063	4.073	1.990	2.300	3.763	4.077	4.075	2.002
1828	20,558	4.313	4.079	2.234	2.410	3.903	4.071	4.106	1.965
1829	24,611	4.391	4.077	2.314	2.338	4.053	4.057	4.147	1.910
1830	38,200	4.582	4.085	2.496	2.118	4.464	4.060	4.166	1.895
1831	16,347	4.213	4.101	2.112	1.825	4.388	4.097	4.176	1.921
1832	7,317	3.864	4.147	1.717	1.638	4.226	4.146	4.167	1.979
1833	4,938	3.694	4.185	1.509	1.600	4.094	4.206	4.175	2.031
1834	6,171	3.790	4.233	1.557	1.730	4.060	4.241	4.180	2.061
1835	9,999	4.000	4.230	1.770	1.930	4.070	4.295	4.181	2.114
1836	30,016	4.477	4.279	2.198	2.196	4.281	4.278	4.191	2.087
1837	45,152	4.655	4.294	2.360	2.348	4.307	4.271	4.209	2.062
1838	66,691	4.824	4.319	2.505	2.404	4.420	4.270	4.224	2.046
1839	35,843	4.554	4.326	2.225	2.262	4.292	4.280	4.237	2.043
1840	45,143	4.655	4.328	2.326	2.008	4.647	4.300	4.244	2.056
1841	10,034	4.002	4.311	1.690	1.728	4.274	4.314	4.244	2.070
1842	8,247	3.916	4.293	1.623	1.604	4.312	4.314	4.226	2.088
1843	7,173	3.856	4.276	1.579	1.643	4.213	4.311	4.217	2.094
1844	10,359	4.015	4.286	1.729	1.810	4.206	4.285	4.217	2.068

Year	Index	Logarithms	9-Year Moving Average	Deviations from 9-Year Moving Average	Ideal 9.6-Year Cycle	Logs of Residual	11-Year Moving Average of Residual	23-Year Moving Average of Residual	Deviations of 11-Year from 23-Year Moving Average
1845	21,180	4.326	4.248	2.078	2.042	4.284	4.262	4.220	2.042
1846	31,062	4.492	4.240	2.252	2.268	4.224	4.210	4.223	1.987
1847	47,065	4.673	4.219	2.454	2.397	4.276	4.190	4.223	1.967
1848	43,735	4.641	4.197	2.444	2.366	4.275	4.167	4.216	1.951
1849	20,353	4.309	4.168	2.140	2.176	4.133	4.163	4.206	1.957
1850	8,519	3.930	4.138	1.792	1.888	4.042	4.165	4.180	1.985
1851	5,361	3.729	4.124	1.605	1.662	4.067	4.155	4.171	1.984
1852	9,552	3.658	4.105	1.553	1.595	4.063	4.145	4.148	1.997
1853	5,682	3.755	4.093	1.662	1.700	4.054	4.126	4.146	1.980
1854	11,358	4.055	4.099	1.956	1.885	4.170	4.112	4.146	1.966
1855	23,362	4.367	4.127	2.242	2.145	4.223	4.108	4.159	1.949
1856	31,642	4.500	4.142	2.359	2.325	4.175	4.089	4.170	1.919
1857	33,757	4.528	4.140	2.388	2.410	4.118	4.089	4.173	1.916
1858	23,226	4.366	4.134	2.232	2.300	4.066	4.094	4.174	1.920
1859	15,178	4.181	4.098	2.083	2.062	4.119	4.110	4.171	1.939
1860	7,272	3.862	4.081	1.780	1.773	4.089	4.124	4.164	1.960
1861	4,448	3.648	4.087	1.561	1.818	3.830	4.150	4.166	1.984
1862	4,926	3.693	4.127	1.566	1.620	4.072	4.174	4.173	2.001
1863	5,437	3.735	4.179	1.556	1.617	4.118	4.196	4.178	2.018
1864	16,498	4.217	4.223	1.995	1.988	4.229	4.213	4.184	2.029
1865	35,971	4.556	4.260	2.296	2.232	4.324	4.221	4.191	2.030
1866	76,556	4.884	4.288	2.596	2.375	4.509	4.223	4.188	2.035
1867	68,392	4.835	4.290	2.545	2.390	4.445	4.255	4.184	2.071
1868	37,447	4.573	4.302	2.272	2.219	4.354	4.268	4.192	2.076
1869	15,686	4.196	4.283	1.912	1.942	4.254	4.273	4.202	2.071
1870	7,942	3.900	4.253	1.647	1.690	4.210	4.269	4.211	2.058
1871	5,123	3.710	4.208	1.501	1.598	4.112	4.260	4.213	2.047
1872	7,106	3.852	4.186	1.665	1.670	4.182	4.223	4.231	1.992
1873	11,250	4.051	4.170	1.881	1.845	4.206	4.194	4.261	1.933
1874	18,774	4.274	4.177	2.097	2.092	4.182	4.195	4.281	1.914
1875	30,508	4.484	4.209	2.276	2.300	4.184	4.203	4.297	1.906
1876	42,834	4.632	4.238	2.394	2.410	4.222	4.209	4.299	1.910
1877	27,345	4.437	4.241	2.196	2.338	4.099	4.215	4.296	1.919
1878	17,834	4.251	4.225	2.026	2.118	4.133	4.244	4.289	1.955
1879	15,386	4.187	4.243	1.944	1.825	4.362	4.272	4.283	1.989
1880	9,443	3.975	4.268	1.707	1.638	4.337	4.303	4.281	2.022
1881	7,599	3.881	4.295	1.586	1.600	4.281	4.331	4.283	2.048
1882	8,061	3.906	4.346	1.560	1.730	4.176	4.335	4.285	2.050
1883	27,187	4.434	4.377	2.058	1.930	4.504	4.351	2.293	2.058
1884	51,511	4.712	4.387	2.325	2.196	4.516	4.369	4.301	2.068
1885	74,050	4.870	4.396	2.473	2.348	4.522	4.365	4.302	2.063
1886	78,773	4.896	4.401	2.496	2.404	4.492	4.361	4.305	2.056
1887	33,899	4.530	4.404	2.126	2.262	4.268	4.363	3.310	2.053
1888	18,886	4.276	4.368	1.908	2.008	4.268	4.371	4.299	2.072
1889	11,520	4.062	4.325	1.738	1.728	4.334	4.353	4.302	2.051
1890	8,552	3.992	4.290	1.632	1.604	4.318	4.338	4.308	2.030
1891	8,660	3.938	4.274	1.664	1.643	4.295	4.311	4.310	2.001
1892	12,902	4.111	4.281	1.830	1.810	4.301	4.290	4.313	1.977
1893	20,331	4.308	4.298	2.010	2.042	4.266	4.292	4.320	1.972
1894	36,853	4.566	4.311	2.255	2.268	4.299	4.266	4.329	1.937
1895	56,407	4.751	4.281	2.470	2.397	4.354	4.261	4.319	1.932
1896	39,437	4.596	4.262	2.334	2.366	4.230	4.246	4.294	1.952
1897	26,761	4.428	4.245	2.183	2.176	4.252	4.256	4.261	1.995
1898	15,185	4.181	4.242	1.939	1.888	4.293	4.266	4.237	2.029
1899	4,473	3.651	4.241	1.409	1.662	3.989	4.282	4.233	2.049
1900	5,781	3.762	4.243	1.519	1.595	4.167	4.289	4.235	2.054
1901	9,117	3.960	4.265	1.695	1.700	4.260	4.283	4.236	2.047
1902	19,267	4.285	4.282	2.002	1.885	4.400	4.255	4.234	2.021
1903	36,116	4.558	4.261	2.297	2.145	4.413	4.211	4.220	1.991
1904	58,850	4.770	4.248	2.522	2.325	4.445	4.180	4.196	1.984
1905	61,478	4.789	4.227	2.562	2.410	4.379	4.196	4.184	2.012
1906	38,501	4.586	4.207	2.379	2.300	4.286	4.209	4.162	2.047
1907	9,704	3.987	4.184	1.803	2.062	3.925	4.219	4.133	2.086
1908	3,410	3.533	4.162	1.371	1.773	3.760	4.206	4.107	2.099
1909	3,774	3.577	4.131	1.446	1.620	3.957	4.166	4.085	2.081
1910	6,000	3.778	4.082	1.696	1.617	4.161	4.103	4.064	2.039
1911	12,000	4.079	4.033	2.046	1.766	4.313	4.068	4.051	2.017
1912	22,877	4.359	4.058	2.302	1.988	4.371	4.025	4.032	1.993
1913	30,991	4.491	4.082	2.410	2.232	4.259	4.002	4.005	1.997
1914	22,404	4.350	4.058	2.292	2.375	3.975	3.990	3.982	2.008
1915	13,817	4.140	3.997	2.143	2.390	3.750	3.970	3.961	2.009
1916	16,259	4.211	3.923	2.288	2.219	3.992	3.939	3.946	1.993
1917	5,606	3.749	3.846	1.903	1.942	3.807	3.882	3.931	1.951
1918	2,325	3.366	3.767	1.600	1.690	3.676	3.824	3.913	1.911
1919	1,695	3.229	3.731	1.498	1.598	3.631	3.768	3.907	1.861
1920	2,571	3.410	3.721	1.689	1.670	3.740	3.759	3.899	1.860
1921	4,599	3.663	3.721	1.942	1.845	3.818	3.776	3.879	1.897
1922	6,028	3.780	3.755	2.025	2.092	3.688	3.786	3.849	1.937
1923	10,682	4.029	3.817	2.212	2.300	3.729	3.805	3.815	1.990
1924	11,349	4.055	3.865	2.190	2.410	3.645	3.824	3.785	2.039
1925	16,106	4.207	3.873	2.334	2.338	3.869	3.836	3.767	2.069
1926	11,352	4.055	3.855	2.200	2.118	3.937	3.823	3.763	2.060
1927	8,451	3.927	3.826	2.101	1.825	4.102	3.792	3.770	2.022
1928	4,589	3.661	3.787	1.875	1.638	4.024	3.773	3.762	2.011
1929	3,014	3.479	3.761	1.718	1.600	3.879	3.755	3.754	2.001
1930	3,154	3.499	3.731	1.768	1.730	3.769	3.759	3.742	2.017
1931	3,343	3.524	3.720	1.805	1.930	3.594	3.756	3.724	2.032
1932	4,719	3.677	3.710	1.964	2.196	3.478	3.752	3.701	2.051
1933	6,643	3.822	3.705	2.117	2.348	3.474	3.733	3.679	2.054
1934	8,706	3.940	3.708	2.232	2.404	3.536	3.716	3.666	2.050
1935	8,880	3.948	3.705	2.243	2.262	3.686	3.691	3.662	2.029
1936	6,936	3.841	3.694	2.148	2.008	3.833	3.658	3.666	1.992
1937	4,167	3.620	3.668	1.951	1.728	3.892	3.623	3.655	1.968
1938	3,207	3.506	3.630	1.876	1.604	3.902	3.599	3.635	1.964
1939	2,955	3.471	3.593	1.878	1.643	3.828	3.585	3.617	1.968
1940	2,627	3.420	3.564	1.855	1.810	3.610	3.572*	3.600	1.972
1941	2,811	3.449	3.534	1.915	2.042	3.407	3.566*	3.587	1.979
1942	3,008	3.478	3.521	1.957	2.268	3.210	3.559*	3.574	1.985
1943	4,033	3.600	3.511	2.094	2.397	3.209	3.534*	3.560	1.974
1944	4,876	3.688	3.482	2.206	2.366	3.322	3.494*	3.547	1.947
1945	3,693	3.567	3.455	2.113	2.176	3.391	3.482*	3.534	1.948
1946	3,223	3.508	3.470	2.039	1.888	3.620		3.521	
1947	2,603	3.416			1.662	3.754		3.508	
1948	1,625	3.211			1.595	3.616		3.494	
1949	1,476	3.169			1.700	3.469		3.481	
1950	3,821	3.582			1.885	3.697		3.468	
								3.455	

TABLE 2

Clearspan Numbers of Lynx. Deviations of logarithms from their 9-year moving average. An asterisk * denotes red clearspan (clearspan numbers of declining values).

Year	Deviation	Clearspan	Year	Deviation	Clearspan	Year	Deviation	Clearspan
1739	1.803	-----	1774	1.490	17*	1809	2.475	70+
1740	1.945	1+	1775	1.732	1	1810	2.426	1*
1741	2.018	2+	1776	2.132	4	1811	2.414	2*
1742	2.005	1*	1777	2.150	5	1812	1.462	55*
1743	2.085	5+	1778	2.203	6	1813	1.396	74 *
1744	2.032	1*	1779	2.140	2*	1814	1.469	2
1745	1.996	5*	1780	2.208	8	1815	1.411	1*
1746	1.817	6*	1781	2.018	5*	1816	1.849	4
1747	1.976	1	1782	1.836	6*	1817	2.157	5
1748	1.928	1*	1783	1.859	1	1818	2.332	6
1749	1.919	2*	1784	1.808	8*	1819	2.534	80+
1750	2.084	6	1785	1.820	1	1820	2.629	81+
1751	2.236	12+	1786	1.979	5	1821	1.918	4*
1752	2.418	13+	1787	2.237	15	1822	1.830	6*
1753	2.343	1*	1788	2.094	1*	1823	1.809	7*
1754	1.875	7*	1789	1.882	4*	1824	1.718	8*
1755	1.692	16+*	1790	2.083	1	1825	1.750	1
1756	1.422	17+*	1791	2.021	1*	1826	1.828	3
1757	1.657	1	1792	2.086	3	1827	1.990	6
1758	1.917	4	1793	1.894	3*	1828	2.234	8
1759	2.227	5	1794	1.885	4*	1829	2.314	9
1760	2.336	6	1795	1.870	9*	1830	2.496	10
1761	2.341	7	1796	1.866	10*	1831	2.112	3*
1762	2.347	9	1797	1.876	2	1832	1.717	16*
1763	2.050	4*	1798	2.131	17	1833	1.508	17*
1764	1.854	5*	1799	2.244	27	1834	1.557	1
1765	1.678	7*	1800	2.341	28	1835	1.770	3
1766	1.671	8*	1801	2.262	1*	1836	2.198	5
1767	1.693	3	1802	2.014	4*	1837	2.360	6
1768	2.050	4	1803	1.735	27*	1838	2.506	17
1769	2.265	6	1804	1.612	29*	1839	2.228	1*
1770	2.312	7	1805	1.730	1	1840	2.326	2
1771	2.410	18	1806	1.926	3	1841	1.690	6*
1772	1.991	4*	1807	2.175	5	1842	1.623	7*
1773	1.862	5*	1808	2.383	36	1843	1.579	8*

Year	Deviation	Clearspan	Year	Deviation	Clearspan	Year	Deviation	Clearspan
1844	1.729	3	1879	1.944	5*	1914	2.292	2*
1845	2.078	4	1880	1.707	8*	1915	2.143	4*
1846	2.252	5	1881	1.586	10*	1916	2.288	1
1847	2.454	8	1882	1.560	11*	1917	1.903	5*
1848	2.444	1*	1883	2.058	5	1918	1.600	8*
1849	2.140	3*	1884	2.325	7	1919	1.498	9*
1850	1.792	5*	1885	2.473	17	1920	1.689	1
1851	1.605	7*	1886	2.496	18	1921	1.942	4
1852	1.553	18*	1887	2.126	3*	1922	2.025	5
1853	1.662	2	1888	1.908	5*	1923	2.212	6
1854	1.956	4	1889	1.738	6*	1924	2.190	1*
1855	2.242	6	1890	1.632	7*	1925	2.334	18
1856	2.359	7	1891	1.664	1	1926	2.200	1*
1857	2.388	8	1892	1.830	3	1927	2.101	4*
1858	2.232	3*	1893	2.010	5	1928	1.875	7*
1859	2.083	4*	1894	2.255	7	1929	1.718	8*
1860	1.780	6*	1895	2.470	8	1930	1.768	1
1861	1.561	8*	1896	2.334	1*	1931	1.805	2
1862	1.566	1	1897	2.183	3*	1932	1.964	4
1863	1.556	10*	1898	1.939	5*	1933	2.117	6
1864	1.995	4	1899	1.409	85*	1934	2.232	7
1865	2.296	7	1900	1.519	1	1935	2.243	8
1866	2.596	46	1901	1.695	2	1936	2.148	2*
1867	2.545	1*	1902	2.002	4	1937	1.951	5*
1868	2.272	4*	1903	2.297	6	1938	1.876	6*
1869	1.912	6*	1904	2.522	36	1939	1.877	7*
1870	1.647	7*	1905	2.562	38	1940	1.855	8*
1871	1.501	55*	1906	2.379	2*	1941	1.915	3
1872	1.665	2	1907	1.803	5*	1942	1.957	5
1873	1.881	3	1908	1.371	169+*	1943	2.094	6
1874	2.097	5	1909	1.446	1	1944	2.206	8
1875	2.276	7	1910	1.696	2	1945	2.113	1*
1876	2.394	8	1911	2.046	4	1946	2.039	3*
1877	2.196	2*	1912	2.302	5			
1878	2.026	4*	1913	2.410	8			

TABLE 3

Lynx Clearspan Numbers from a 5-Cycle, 8-Section Average after 9.6, 9.0, and 7.46-year Cycle Removed. An (*) Denotes Clearspan of Declining Value. (Red Clearspan)

Year	Clearspan	Year	Clearspan	Year	Clearspan	Year	Clearspan	Year	Clearspan
1754	- -	1790	2	1826	1	1862	5	1898	5*
1755	- -	1791	5	1827	2	1863	6	1899	7*
1756	1+*	1792	30	1828	4*	1864	7	1900	1
1757	2+*	1793	1*	1829	7*	1865	4*	1901	3
1758	3+*	1794	2*	1830	2	1866	23*	1902	4
1759	3	1795	2	1831	1*	1867	1	1903	5
1760	5+	1796	21*	1832	7	1868	3	1904	6
1761	6+	1797	38*	1833	1*	1869	4	1905	7
1762	2*	1798	2	1834	26*	1870	5	1906	3*
1763	3*	1799	1*	1835	2	1871	1*	1907	8*
1764	5*	1800	4	1836	3	1872	7	1908	1
1765	1	1801	1*	1837	1*	1873	3*	1909	3
1766	7*	1802	9	1838	34	1874	7*	1910	4
1767	3	1803	48+	1839	1*	1875	1	1911	161
1768	6	1804	4*	1840	2*	1876	1*	1912	1*
1769	1*	1805	7*	1841	6*	1877	3	1913	2*
1770	2*	1806	1	1842	44	1878	5	1914	5*
1771	1	1807	1*	1843	2	1879	6	1915	72*
1772	5*	1808	4	1844	3	1880	7	1916	7
1773	6*	1809	1*	1845	4	1881	3*	1917	2
1774	7*	1810	1	1846	43	1882	7*	1918	4
1775	3	1811	7	1847	2*	1883	16*	1919	6
1776	7	1812	4*	1848	1	1884	2	1920	23
1777	1*	1813	5*	1849	5*	1885	3	1921	1*
1878	2*	1814	2	1850	0	1886	5	1922	2*
1779	1	1815	1*	1851	2	1887	22	1923	6*
1780	5*	1816	1	1852	3	1888	1*	1924	7*
1781	6*	1817	1*	1853	4	1889	3*	1925	2
1782	1	1818	4*	1854	5	1890	6*	1926	1*
1883	3	1819	7	1855	3*	1891	24*	1927	4
1784	7	1820	1*	1856	9	1892	1	1928	5
1785	1*	1821	7*	1857	6*	1893	3	1929	1*
1786	2*	1822	2	1858	15*	1894	5	1930	3*
1787	1	1823	4*	1859	1	1895	97		
1788	5*	1824	20	1860	2	1896	98		
1789	7*	1825	1*	1861	4	1897	2*		

TABLE 4

9.6 Year Periodic Table of Lynx Deviations of Logarithms from the 9-year moving average. Figures in Column 9.6 denoted by an Asterisk (*) appears also in the first Column of the succeeding Base Cycle. (The Original Table used four places logarithms that are here approximated to three places).

Outfit	1	2	3	4	5	6	7	8	9	9.6
1738	1.803	1.945	2.018	2.005	2.085	2.032	1.996	1.817	1.976	1.928
1748	1.919	2.084	2.236	2.418	2.343	1.875	1.692	1.422	1.657	1.917
1757	1.917	2.227	2.336	2.341	2.347	2.050	1.854	1.678	1.671	1.693
1767	2.050	2.265	2.312	2.410	1.991	1.862	1.490	1.732	2.132	2.150*
1776	2.150	2.203	2.140	2.208	2.018	1.836	1.859	1.808	1.820	1.979
1786	2.237	2.094	1.882	2.083	2.021	2.086	1.894	1.885	1.870	1.866
1796	1.876	2.131	2.244	2.342	2.262	2.014	1.735	1.612	1.730	1.926*
1805	1.926	2.175	2.383	2.475	2.426	2.414	1.462	1.396	1.469	1.411
1815	1.849	2.157	2.332	2.535	2.556	1.918	1.830	1.809	1.718	1.777*
1824	1.777	1.828	1.990	2.234	2.314	2.496	2.112	1.717	1.508	1.557
1834	1.770	2.198	2.360	2.505	2.228	2.326	1.690	1.623	1.579	1.729
1844	2.078	2.252	2.454	2.444	2.140	1.792	1.605	1.553	1.662	1.956*
1853	1.956	2.242	2.359	2.388	2.232	2.083	1.780	1.561	1.566	1.556
1863	1.995	2.296	2.596	2.545	2.272	1.912	1.647	1.501	1.665	1.881*
1872	1.881	2.097	2.276	2.394	2.196	2.026	1.944	1.707	1.586	1.561
1882	2.058	2.325	2.473	2.496	2.126	1.908	1.738	1.632	1.664	1.830
1892	2.010	2.255	2.470	2.334	2.183	1.939	1.409	1.519	1.695	2.002*
1901	2.002	2.297	2.522	2.562	2.379	1.803	1.371	1.446	1.696	2.046
1911	2.302	2.410	2.292	2.143	2.288	1.903	1.600	1.498	1.689	1.942*
1920	1.942	2.025	2.212	2.190	2.334	2.200	2.101	1.875	1.718	1.768
1930	1.805	1.964	2.117	2.232	2.243	2.148	1.951	1.876	1.872	1.855
1940	1.915	1.957	2.094	2.206	2.113	2.039				

Average Medians for Sections of Table:

1738-1872	1.924	2.170	2.299	2.376	2.235	2.005	1.777	1.661	1.670	1.827
1805-1940	1.945	2.197	2.351	2.395	2.253	2.009	1.715	1.601	1.665	1.820
1738-1940	1.935	2.183	2.325	2.385	2.244	2.007	1.746	1.631	1.668	1.823

TABLE 5

5-year Periodic Table of Regular 5 and 6-year Cycles of Numbers. Averaging the Table by Six-Cycle Sections Neutralize the 6-year Cycle and reveals the 5-year Cycle.

Base Year	Years After Base Year 1	2	3	4	5
0	2	4	6	7	5
5	3	3	5	6	6
10	4	4	4	5	5
15	5	5	5	4	4
20	4	6	6	5	3
25	3	5	7	6	4
30	2	4	6	7	5
35	3	3	5	6	6
40	4	4	4	5	5
45	5	6	5	4	4

A. Average of Table by 5-line Sections (0-20, 25-45)

Total (0-20)	18	22	26	27	23
Average	3.6	4.4	5.2	5.2	4.6
Total (25-45)	17	21	27	28	24
Average	3.4	4.2	5.4	5.3	5.8

B. Average of Table by 6-line Overlapping Sections (0-25, 20-45)

Total (0-25)	21	27	33	33	27
Average	3.5	4.5	5.5	5.5	4.5
Less 6-year Cycle Average (2.5) = 5-year Cycle	1.0	2.0	3.0	3.0	2.0
Total (20-45)	21	27	33	33	27
Average	3.5	4.5	5.5	5.5	4.5
Less 6-year Cycle Average (2.5) = 5-year Cycle	1.0	2.0	3.0	3.0	2.0

TABLE 6

6-year Periodic Table of Regular 5 and 6 Year Cycle of Numbers. Averaging the Table by 10-Cycle sections neutralizes the 5-year Cycle and reveals the 6-year Cycle.

Base Year	1	2	3	4	5	6
0	2	4	6	7	5	3
6	3	5	6	6	4	4
12	4	5	5	5	5	5
18	4	4	4	6	6	5
24	3	3	5	7	6	4
30	2	4	6	7	5	3
36	3	5	6	6	4	4
42	4	5	5	5	5	5
48	4	4	4	6	6	5
54	3	3	5	7	6	4
60	2	4	6	7	5	3
66	3	5	6	6	4	4

A. Average of Table by 6-line sections (0-30, 36-66)

Total (0-30)	18	25	32	38	31	24
Average	3.0	4.2	5.3	6.3	5.2	4.0
Total (36-66)	19	26	32	37	30	25
Average	3.2	4.3	5.3	6.2	5.0	4.1

B. Average of Table by 10-line overlapping sections, (0-54, 12-66)

Total (0-54)	32	42	52	62	52	4 2
Average	3.2	4.2	5.2	6.2	5.2	4.2
Less 5-year Cycle Average (2.2) = 6-year Cycle	1.0	2.0	3.0	4.0	3.0	2.0
Total (36-66)	32	42	52	62	52	42
Average	3.2	4.2	5.2	6.2	5.2	4.2
Less 5-year Cycle average (2.2) = 6-year Cycle	1.0	2.0	3.0	4.0	3.0	2.0

TABLE 7

7.9-Year Periodic Table of Lynx Deviations of Logarithms from 9-year moving average after 9.6, 9.0 and 7.46-year cycles have been subtracted. The numbers of Column 7.9 indicated by an Asterisk (*) appears also in Column 1 of the succeeding Cycle.

Year	1	2	3	4	5	6	7	7.9
1738	1.830	1.864	1.553	1.540	1.872	2.146	2.395	2.257
1746	2.308	2.009	1.868	1.798	1.931	2.110	2.202	1.988
1754	2.008	1.841	1.978	1.913	1.994	2.024	2.008	2.161
1762	2.104	2.112	2.057	1.997	1.846	1.980	2.070	2.008
1770	2.056	1.800	1.906	1.808	2.188	2.415	2.102	1.949
1778	1.800	1.891	1.926	1.923	2.135	2.139	2.072	2.016*
1785	2.016	2.222	1.942	1.678	1.843	1.803	2.079	2.052
1793	2.137	2.099	2.062	2.018	2.037	2.008	1.969	1.983
1801	1.961	1.972	1.925	2.046	2.046	2.050	2.120	2.094
1809	2.193	2.297	1.692	1.684	1.758	1.572	1.996	2.073
1817	2.111	2.206	2.288	1.962	1.971	2.108	2.035	1.909
1825	1.796	1.761	1.881	2.001	2.382	2.217	2.053	1.924
1833	1.848	1.968	2.086	2.035	2.023	1.978	2.216	1.964
1841	2.059	2.000	1.995	2.156	2.034	2.069	2.048	1.840
1849	1.764	1.880	1.953	2.047	2.072	2.018	2.020	1.966*
1856	1.966	1.906	2.033	2.029	1.952	1.996	1.837	2.116
1864	2.075	2.191	2.021	1.949	1.937	2.012	2.015	2.032
1872	1.977	1.845	1.844	1.860	1.918	2.103	2.312	2.196
1880	1.958	1.711	2.017	2.072	2.098	2.123	1.948	1.990
1888	2.081	2.014	1.960	1.947	2.058	2.015	2.143	1.940
1896	1.970	1.985	1.714	1.951	2.014	2.018	2.001	2.127
1904	2.161	2.203	1.938	1.721	1.860	1.990	2.143	2.337
1912	2.147	1.998	1.879	2.189	1.967	1.854	1.861	1.805
1920	1.904	1.862	1.964	1.908	2.210	2.234	2.328	2.139
1928	1.980	1.895	1.882	1.833	1.860	1.908	2.003	2.151
1936	2.199	2.221	2.142	2.020	2.001	1.805	1.822	1.871

Average Medians (11 items) for sections of table and their 3-year moving average.

1738-1864	2.000	1.991	1.965	1.948	1.994	2.017	2.057	2.016
3-year moving average	2.002	1.988	1.968	1.969	1.996	2.033	2.040	2.024
1809-1936	2.021	1.991	1.957	1.974	1.995	2.021	2.037	2.021
3-year moving average	2.011	1.990	1.974	1.975	1.997	2.015	2.026	2.02

TABLE 8

Fifteen Year Periodic Table of Deviations of Logarithms from 9-Year Moving Average. (9.6-year cycle removed)

Year	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1739	1.891	1.886	1.891	1.890	1.892	1.915	1.951	1.969	2.091	2.126	2.137	2.086	2.048	1.994	1.960
1754	1.951	1.960	1.985	1.989	2.006	2.026	2.040	2.049	2.065	2.063	2.033	2.013	1.992	1.975	1.962
1769	1.983	2.037	2.063	2.074	2.056	2.031	2.000	1.976	2.013	2.033	2.067	2.061	2.026	1.967	1.889
1784	1.859	1.845	1.884	1.945	1.983	1.994	1.983	1.953	1.917	1.913	1.966	2.010	2.070	2.095	2.111
1799	2.092	2.090	2.105	2.131	2.166	2.192	2.217	2.159	2.081	1.989	1.905	1.836	1.784	1.756	1.767
1814	1.816	1.830	1.843	1.923	1.967	1.992	2.031	2.050	2.074	2.068	2.068	2.031	2.002	1.965	1.910
1829	1.894	1.921	1.979	2.031	2.061	2.114	2.087	2.062	2.046	2.043	2.056	2.070	2.088	2.094	2.068
1844	2.042	1.987	1.967	1.951	1.957	1.985	1.984	1.997	1.980	1.966	1.949	1.919	1.916	1.920	1.939
1859	1.960	1.984	2.001	2.018	2.029	2.030	2.035	2.071	2.076	2.071	2.058	2.047	1.992	1.933	1.914
1874	1.906	1.910	1.919	1.955	1.989	2.022	2.048	2.050	2.058	2.068	2.063	2.058	2.053	2.072	2.051
1889	2.030	2.001	1.977	1.972	1.937	1.932	1.952	1.995	2.029	2.049	2.054	2.047	2.021	1.991	1.984
1904	2.012	2.047	2.086	2.099	2.081	2.039	2.017	1.993	1.997	2.008	2.009	1.993	1.951	1.911	1.861
1919	1.860	1.897	1.937	1.990	2.039	2.069	2.060	2.022	2.011	2.001	2.017	2.032	2.051	2.054	2.050
1934	2.029	1.992	1.968	1.964	1.968	1.972	1.979	1.985	1.974	1.947	1.948				
1739-1859	1.940	1.946	1.967	1.990	2.008	2.025	2.023	2.023	2.048	2.033	2.028	2.022	2.007	1.978	1.949
1814-1934	1.956	1.956	1.964	1.983	2.002	2.016	2.022	2.025	2.028	2.029	2.029	2.018	1.997	1.978	1.958
1739-1934	1.947	1.951	1.966	1.987	2.005	2.020	2.022	2.024	2.038	2.031	2.029	2.020	2.002	1.978	1.953
Amplitude Restored by factor of 1.89	1.900	1.907	1.936	1.975	2.009	2.038	2.042	2.045	2.072	2.059	2.055	2.038	2.004	1.958	1.911
Value from fitted curve	1.915	1.915	1.928	1.950	1.990	2.030	2.063	2.080	2.088	2.080	2.063	2.030	1.990	1.950	1.928

TABLE 9

6-year Periodic Table of Regular 5 and 6 year Cycle of Numbers.

Base Year	1	2	3	4	5	6
0	2	4	6	7	5	3
6	3	5	6	6	4	4
12	4	5	5	5	5	5
18	4	4	4	6	6	5
24	3	3	5	7	6	4
30	2	4	6	7	5	3
36	3	5	6	6	4	4
42	4	5	5	5	5	5
48	4	4	4	6	6	5
54	3	3	5	7	6	4
60	2	4	6	7	5	3
66	3	5	6	6	4	4

Less 6-year Cycle (1,2,3,4,3,2) reveals 5-year Cycle.

0	1	2	3	3	2	1
6	2	3	3	2	1	2
12	3	3	2	1	2	3
18	3	2	1	2	3	3
24	2	1	2	3	3	2
30	1	2	3	3	2	1
36	2	3	3	2	1	2
42	3	3	2	1	2	3
48	3	2	1	2	3	3
54	2	1	2	3	3	2
60	1	2	3	3	2	1
66	2	3	3	2	1	2

Less 5-year Cycle (1,2,3,3,2) reveals 6-year Cycle.

0	1	2	3	4	3	2
6	1	2	3	4	3	2
12	1	2	3	4	3	2
18	1	2	3	4	3	2
24	1	2	3	4	3	2
30	1	2	3	4	3	2
36	1	2	3	4	3	2
42	1	2	3	4	3	2
48	1	2	3	4	3	2
54	1	2	3	4	3	2
60	1	2	3	4	3	2
66	1	2	3	4	3	2

TABLE 11

Probable Amplitude of Eight Lynx deviations as Percent of the Trend.

Cycle Length	7.46 years	7.95 years	9.0 years	9.6 years	11.75 years	15.05 years	22.2 years	35.2 years
Percent at High	113.8	112.5	126.6	257.0	134.9	122.5	138.0	144.5
Percent at Low	87.9	81.7	79.1	38.9	74.1	81.7	72.4	69.2

TABLE 10

Calendar Years for Highs and Lows of Periodicities of Lengths Shown. (An asterisk * denotes an adjustment for decimal portion of Year Accumulating by fitting 8-year curve for 7.95, 22-year curve for 22.2 and 35-year curve for 35.2 as explained in the text.)

Probable Calendar Years of High

7.46 years	7.95 years	9.0 years	9.6 years	11.75 years	15.05 years	22.2 years	35.2 years
1742	1737	1741	1742	1736	1748	1754	1767
1749	1745	1750	1751	1748	1763	1776	1802
1757	1753	1759	1761	1760	1778	1798	1837
1764	1761	1768	1771	1771	1793	1820	1873
1772	1769	1777	1780	1783	1808	1842	1907
1779	1777	1786	1790	1795	1823	1864	1943*
1786	1785	1795	1799	1807	1838	1886	1978
1794	1793	1804	1809	1818	1853	1909*	
1801	1801	1813	1819	1830	1868	1931	
1809	1809	1822	1828	1842	1883	1953	
1816	1817	1831	1838	1854	1898		
1824	1825	1840	1847	1865	1913		
1831	1832	1849	1857	1877	1928		
1839	1840	1858	1867	1889	1943		
1846	1848	1867	1876	1901	1958		
1854	1856	1876	1886	1912			
1861	1864	1885	1895	1924			
1869	1872	1894	1905	1936			
1876	1880	1903	1915	1948			
1883	1888	1912	1924	1959			
1891	1896	1921	1934				
1898	1904	1930	1943				
1906	1912	1939	1953				
1913	1920	1948					
1921	1928						
1928	1936						
1936	1943*						
1943	1951						
1951							

Probable Calendar Years of Lows

1738	1741	1736	1737	1742	1740	1742	1749
1746	1749	1745	1746	1754	1756	1765	1785*
1753	1757	1754	1756	1766	1771	1787	1820
1760	1765	1763	1766	1777	1786	1809	1855
1768	1773	1722	1775	1789	1801	1831	1890
1775	1781	1781	1785	1801	1816	1853	1925
1783	1789	1790	1794	1812	1831	1875	1961*
1790	1797	1799	1804	1824	1846	1897	
1798	1805	1808	1814	1836	1861	1919	
1805	1813	1817	1823	1848	1876	1941	
1813	1821	1826	1833	1860	1891	1964*	
1820	1828	1835	1842	1871	1906		
1828	1836	1844	1852	1883	1921		
1835	1844	1853	1862	1895	1936		
1842	1852	1862	1871	1906	1951		
1850	1860	1871	1881	1918	1966		
1857	1868	1880	1890	1930			
1865	1876	1889	1900	1942			
1872	1884	1898	1910	1954			
1880	1892	1907	1919				
1887	1900	1916	1929				
1895	1908	1925	1938				
1902	1916	1934	1948				
1910	1924	1943	1958				
1917	1932	1952					
1925	1940						
1932	1948						
1939	1955*						
1947							
1954							

TABLE 13

A. Intervals of Manifest Cycle of Each Derived Cycles (7.46, 7.95, 9.0, 9.6, 11.75, 15.05, 22.2, and 35.2 years).

Year of High	Interval (years)	Year of Low	Interval (years)
1741		1738	
1751	10	1745	7
1761	10	1755	10
1771	10	1765	10
1779	8	1775	10
1794	14	1785	10
1800	7	1794	9
1809	9	1804	10
1819	10	1813	9
1829	10	1823	10
1839	10	1833	10
1847	8	1843	10
1856	9	1852	9
1867	11	1861	9
1876	9	1871	10
1886	10	1881	10
1896	10	1891	10
1904	8	1900	9
1913	9	1909	9
1925	12	1918	9
1935	10	1929	11
1944	9	1939	10
1952	-	1948	9

B. Frequency of Highs and Lows of Table 13

Interval	Number	Interval	Number
7	1	7	1
8	3	8	1
9	5	9	6
10	9	10	13
11	1	11	1
12	1		
13	0		
14	1		
Total	21		22
Average	9.67		9.55
Median	10		10

TABLE 12

Logarithms of derivative cycles of lynx abundance, manifest cycle of eight derivatives, and residual of lynx index after removal of derivatives.

Year	Logarithms of Index	Logarithms of 746-Year Cycle	Logarithms of 795-Year Cycle	Logarithms of 90-Year Cycle	Logarithms of 96-Year Cycle	Logarithms of 11.75-Year Cycle	Logarithms of 15.05-Year Cycle	Logarithms of 22.2-Year Cycle	Logarithms of 35.2-Year Cycle	Logarithms of Manifest Cycle	Logarithms of Residual
1735	4.414	2.043	2.003	1.931	1.825	2.112	2.063	2.090	2.147	2.214	4.200
1736	4.187	2.004	2.073	1.900	1.638	2.129	2.030	2.059	2.130	1.963	4.224
1737	4.068	1.963	2.088	1.949	1.600	2.120	1.990	2.023	2.113	1.846	4.222
1738	3.976	1.944	2.041	1.989	1.730	2.080	1.950	1.977	2.105	1.816	4.160
1739	3.891	1.965	1.991	2.058	1.930	2.008	1.928	1.941	2.066	1.887	4.004
1740	4.018	2.008	1.933	2.069	2.196	1.930	1.915	1.910	2.030	1.991	4.027
1741	4.088	2.045	1.918	2.102	2.348	1.890	1.915	1.885	1.988	2.091	3.997
1742	4.071	2.054	1.947	2.032	2.404	1.870	1.928	1.865	1.950	2.050	4.021
1743	4.182	2.025	2.003	1.968	2.262	1.885	1.950	1.860	1.920	1.873	4.309
1744	4.171	1.979	2.073	1.931	2.008	1.925	1.990	1.865	1.895	1.666	4.505
1745	4.168	1.948	2.088	1.900	1.728	2.005	2.030	1.885	1.876	1.460	4.708
1746	4.031	1.951	2.041	1.949	1.604	2.083	2.063	1.910	1.864	1.465	4.566
1747	4.252	1.986	1.991	1.989	1.643	2.120	2.080	1.941	1.858	1.608	4.644
1748	4.267	2.031	1.933	2.058	1.810	2.130	2.088	1.977	1.846	1.873	4.394
1749	4.311	2.055	1.918	2.069	2.042	2.114	2.080	2.023	1.842	2.143	4.168
1750	4.476	2.041	1.947	2.102	2.268	2.060	2.063	2.059	1.842	2.382	4.094
1751	4.625	2.001	2.003	2.032	2.397	1.979	2.030	2.090	1.846	2.378	4.247
1752	4.751	1.960	2.073	1.968	2.366	1.917	1.990	2.115	1.858	2.247	4.504
1753	4.645	1.944	2.088	1.931	2.176	1.882	1.950	2.135	1.864	1.970	4.675
1754	4.171	1.968	2.041	1.900	1.888	1.871	1.928	2.140	1.876	1.612	4.559
1755	4.002	2.012	1.991	1.949	1.662	1.894	1.915	2.135	1.895	1.453	4.549
1756	3.746	2.048	1.967	1.989	1.595	1.942	1.915	2.115	1.920	1.457	4.289
1757	3.985	2.053	1.918	2.058	1.700	2.030	1.928	2.090	1.950	1.727	4.258
1758	4.262	2.022	1.947	2.069	1.885	2.094	1.950	2.059	1.987	2.013	4.249
1759	4.606	1.976	2.003	2.102	2.145	2.124	1.990	2.023	2.032	2.395	4.211
1760	4.746	1.947	2.073	2.032	2.325	2.128	2.030	1.977	2.044	2.578	4.168
1761	4.787	1.953	2.088	1.968	2.410	2.105	2.063	1.941	2.095	2.623	4.164
1762	4.801	1.990	2.041	1.931	2.300	2.060	2.080	1.910	2.113	2.425	4.376
1763	4.483	2.033	1.991	1.900	2.062	1.960	2.088	1.885	2.130	2.049	4.434
1764	4.267	2.056	1.933	1.949	1.773	1.908	2.080	1.865	2.147	1.711	4.556
1765	4.078	2.039	1.918	1.989	1.620	1.873	2.063	1.860	2.152	1.514	4.564
1766	4.058	1.996	1.947	2.058	1.617	1.873	2.030	1.865	2.158	1.544	4.514
1767	4.075	1.958	2.003	2.069	1.766	1.904	1.990	1.885	2.160	1.735	4.340
1768	4.421	1.944	2.073	2.102	1.988	1.959	1.950	1.990	2.158	2.064	4.357
1769	4.635	1.970	2.088	2.032	2.232	2.050	1.928	1.941	2.152	2.393	4.242
1770	4.658	2.015	2.041	1.968	2.375	2.105	1.915	1.977	2.147	2.543	4.115
1771	4.760	2.049	1.991	1.931	2.390	2.128	1.915	2.023	2.130	2.557	4.203
1772	4.383	2.051	1.933	1.900	2.219	2.125	1.928	2.059	2.113	2.328	4.055
1773	4.259	2.019	1.918	1.949	1.942	2.094	1.950	2.090	2.105	2.067	4.192
1774	3.871	1.973	1.947	1.989	1.690	2.010	1.990	2.115	2.066	1.780	4.091
1775											
1775	4.089	1.945	2.003	2.058	1.598	1.945	2.030	2.135	2.030	1.744	4.345
1776	4.459	1.955	2.073	2.069	1.670	1.899	2.063	2.140	1.988	1.847	4.612
1777	4.461	1.994	2.088	2.102	1.745	1.871	2.080	2.135	1.950	1.966	4.496
1778	4.486	2.036	2.041	2.032	2.092	1.878	2.088	2.115	1.920	2.202	4.284
1779	4.440	2.056	1.991	1.968	2.300	1.915	2.080	2.090	1.895	2.295	4.145
1780	4.492	2.356	1.933	1.931	2.410	1.980	2.063	2.059	1.876	2.288	4.204
1781	4.240	1.994	1.918	1.900	2.338	2.068	2.030	2.023	1.864	2.133	4.107
1782	4.013	1.955	1.947	1.949	2.118	2.112	1.990	1.977	1.858	1.906	4.107
1783	4.019	1.945	2.003	1.989	1.825	2.129	1.950	1.941	1.846	1.628	4.391
1784	3.943	1.973	2.073	2.058	1.638	2.120	1.928	1.910	1.842	1.542	4.401
1785	3.902	2.018	2.088	2.069	1.600	2.080	1.915	1.885	1.842	1.497	4.405
1786	4.059	2.061	2.041	2.102	1.730	2.008	1.915	1.865	1.846	1.558	4.501
1787	4.334	2.049	1.991	2.032	1.930	1.930	1.928	1.860	1.858	1.578	4.756
1788	4.212	2.015	1.933	1.968	2.196	1.890	1.950	1.865	1.864	1.681	4.531
1789	4.009	1.961	1.918	1.931	2.348	1.870	1.990	1.885	1.876	1.779	4.230
1790	4.227	1.945	1.947	1.900	2.404	1.885	2.030	1.990	1.895	1.916	4.311
1791	4.168	1.957	2.003	1.949	2.262	1.925	2.063	1.941	1.920	2.020	4.148
1792	4.211	1.998	2.073	1.989	2.008	2.006	2.080	1.977	1.950	2.080	4.131
1793	4.017	2.039	2.088	2.058	1.728	2.083	2.088	2.023	1.987	2.094	3.923
1794	4.060	2.056	2.041	2.069	1.604	2.120	2.080	2.059	2.032	2.061	3.999
1795	4.086	2.033	1.991	2.102	1.643	2.130	2.063	2.090	2.044	2.118	3.968
1796	4.138	1.990	1.933	2.032	1.810	2.114	2.030	2.115	2.095	2.119	4.019
1797	4.190	1.953	1.918	1.968	2.042	2.060	1.990	2.135	2.113	2.179	4.011
1798	4.481	1.943	1.947	1.931	2.268	1.929	1.950	2.140	2.130	2.241	4.240
1799	4.594	1.977	2.003	1.900	2.397	1.917	1.928	2.135	2.147	2.404	4.190
1800	4.675	2.022	2.073	1.949	2.366	1.881	1.915	2.115	2.152	2.474	4.201
1801	4.585	2.052	2.088	1.989	2.176	1.871	1.915	2.090	2.158	2.339	4.246
1802	4.341	2.047	2.041	2.058	1.888	1.894	1.928	2.059	2.160	2.075	4.266
1803	4.060	2.012	1.991	2.069	1.662	1.942	1.950	2.023	2.158	1.907	4.153
1804	3.936	1.968	1.933	2.102	1.595	2.030	1.990	1.977	2.152	1.747	4.189
1805	4.044	1.945	1.918	2.032	1.700	2.094	2.030	1.941	2.147	1.807	4.237
1806	4.223	1.960	1.947	1.968	1.885	2.124	2.063	1.910	2.130	1.987	4.236
1807	4.467	2.001	2.003	1.931	2.145	2.128	2.080	1.885	2.113	2.286	4.181
1808	4.584	2.041	2.073	1.900	2.325	2.105	2.088	1.865	2.105	2.502	4.082
1809	4.584	2.055	2.088	1.949	2.410	2.060	2.080	1.860	2.066	2.568	4.016
1810	4.436	2.031	2.041	1.989	2.300	1.960	2.063	1.865	2.030	2.279	4.157
1811	4.297	1.986	1.991	2.058	2.062	1.908	2.030	1.885	1.988	1.908	4.389
1812	3.244	1.951	1.933	2.069	1.773	1.873	1.990	1.990	1.950	1.449	3.795
1813	3.108	1.948	1.918	2.102	1.620	1.873	1.950	1.941	1.920	1.272	3.836
1814	3.139	1.980	1.947	2.032	1.617	1.904	1.928	1.977	1.895	1.280	3.859
1815	3.086	2.025	2.003	1.968	1.766	1.959	1.915	2.023	1.876	1.515	3.571
1816	3.582	2.053	2.073	1.931	1.988	2.050	1.915	2.059	1.864	1.933	3.629
1817	3.949	2.045	2.088	1.900	2.232	2.105	1.928	2.090	1.858	2.246	3.703
1818	4.208	2.008	2.041	1.949	2.375	2.128	1.950	2.115	1.846	2.412	3.796
1819	4.484	1.965	1.991	1.989	2.390	2.125	1.990	2.135	1.842	2.427	4.057
1820	4.638	1.944	1.933	2.058	2.219	2.095	2.030	2.140	1.842	2.260	4.378
1821	3.954	1.963	1.918	2.069	1.942	2.010	2.063	2.135	1.846	1.946	4.008
1822	3.856	2.004	1.947	2.102	1.690	1.945	2.080	2.115	1.858	1.741	4.115
1823	3.810	2.043	2.003	2.032	1.598	1.899	2.088	2.090	1.864	1.617	4.193
1824	3.708	2.054	2.073	1.968	1.670	1.871	2.080	2.059	1.876	1.651	4.057
1825	3.713	2.028	2.088	1.931	1.845	1.878	2.063	2.023	1.895	1.651	4.062
1826	3.861	1.983	1.991	1.900	2.092	1.915	2.030	1.977	1.920	1.808	4.053
1827	4.063	1.949	1.933	1.949	2.300	1.980	1.990	1.941	1.950	1.992	4.071
1828	4.313	1.949	1.918	1.989	2.410	2.068	1.950	1.910	1.987	2.181	4.132
1829	4.391	1.983	1.947	2.058	2.338	2.112	1.928	1.885	2.032	2.283	4.108
1830	4.582	2.028	2.003	2.069	2.118	2.129	1.915	1.866	2.044	2.193	4.389
1831	4.213	2.054	2.073	2.102	1.825	2.120	1.915	1.860	2.095	2.044	4.169
1832	3.864	2.043	2.088	2.032	1.638	2.080	1.928	1.865	2.113	1.787	4.077
1833	3.694	2.004	2.041	1.968	1.600	2.008	1.950	1.885	2.130	1.586	4.108
1834	3.790	1.963	1.991	1.931	1.730	1.930	1.990	1.990	2.147	1.592	4.198
1835	4.000	1.944	1.933	1.900	1.930	1.890	2.030	1.941	2.152	1.838	4.162
1836	4.477	1.965	1.918	1.949	2.196	1.870	2.063	1.977	2.158	2.096	4.381
1837	4.655	2.008	1.947	1.989	2.348	1.885	2.080	2.023	2.160	2.440	4.215
1838	4.824	2.045	2.003	2.058	2.404	1.925	2.088	2.059	2.158	2.740	4.084
1839	4.554	2.054	2.073	2.069	2.262	2.005	2.080	2.090	2.152	2.785	3.769
1840	4.656	2.025	2.088	2.102	2.008	2.083	2.063	2.115	2.147	2.631	4.025
1841	4.001	1.979	2.041	2.032	1.728	2.120	2.030	2.135	2.130	2.195	3.806
1842	3.916	1.948	1.991	1.968	1.604	2.130	1.990	2.140	2.113	1.884	4.032
1843	3.856	1.951	1.993	1.931	1.643	2.114	1.950	2.135	2.105	1.762	4.094
1844	4.015	1.986	1.918	1.900	1.810	2.060	1.928	2.115	2.066	1.783	4.232
1845	4.326	2.031	1.947	1.949	2.042	1.929	1.915	2.090	2.030	1.933	4.393
1846	4.492	2.055	2.003	1.989	2.268	1.917	1.915	2.059	1.988	2.194	4.298
1847	4.673	2.041	2.073	2.058	2.397	1.881	1.928	2.023	1.950	2.352	4.321
1848	4.641	2.001	2.088	2.069	2.366	1.871	1.950	1.977	1.920	2.242	4.399
1849	4.309	1.960	2.041	2.102	2.176	1.894	1.990	1.941	1.895	1.999	4.310
1850	3.930	1.944	1.991	2.032	1.888	1.942	2.030	1.910	1.876	1.613	4.317
1851	3.729	1.968	1.993	1.968	1.662	2.030	2.063	1.885	1.864	1.373	4.356
1852	3.658	2.012	1.918	1.931	1.595	2.094	2.080	1.865	1.858	1.353	4.305
1853	3.754	2.048	1.947	1.900	1.700	2.124	2.088	1.860	1.846	1.513	4.241
1854	4.055	2.053	2.003	1.949	1.885	2.128	2.080	1.865	1.842	1.806	4.250
1855	4.368	2.022	2.073	1.989	2.145	2.105	2.063	1.885	1.842	2.124	4.244
1856	4.500	1.976	2.088	2.058	2.325	2.060	2.030	1.990	1.846	2.293	4.207
1857	4.528	1.947	2.041	2.069	2.410	1.960	1.990	1.941	1.858	2.216	4.312
1858	4.366	1.953	1.991	2.102	2.300	1.908	1.950	1.977	1.864	2.045	4.321
1859	4.181	1.990	1.993	2.032	2.062	1.873	1.928	2.023	1.876	1.717	4.464
1860	3.862	2.033	1.918	1.968	1.773	1.873	1.915	2.059	1.895	1.434	4.428
1861	3.648	2.056	1.947	1.931	1.818	1.904	1.915	2.090	1.920	1.383	4.265
1862	3.692	2.039	2.003	1.900	1.620	1.959	1.928	2.115	1.950	1.491	4.201
1863	3.735	1.996	2.073	1.949	1.617	2.050	1.950	2.135	1.987	1.906	3.829
1864	4.217	1.958	2.088	1.989	1.988	2.105	1.990	2.140	2.032	2.290	3.927
1865	4.556	1.944	2.041	2.058	2.232	2.128	2.030	2.135	2.044	2.634	3.922
1866	4.884	1.970	1.991	2.069	2.375	2.125	2.063	2.115	2.095	2.803	4.081
1867	4.835	2.015	1.943	2.102	2.390	2.095	2.080	2.090	2.113	2.817	4.018
1868	4.573	2.049	1.918	2.032	2.219	2.010	2.088	2.059	2.130	2.505	4.068
1869	4.196	2.051	1.947	1.968	1.942	1.945	2.080	2.023	2.147	2.103	4.093
1870	3.900	2.019	2.003	1.931	1.690	1.899	2.063	1.977	2.152	1.734	4.166
1871	3.710	1.973	2.073	1.900	1.598	1.871	2.030	1.941	2.158	1.544	4.166
1872	3.852	1.945	2.088	1.949	1.670	1.878	1.990	1.910	2.160	1.590	4.262
1873	4.051	1.955	2.041	1.989	1.845	1.915	1.950	1.885	2.158	1.638	4.413
1874	4.274	1.994	1.991	2.058	2.092	1.980	1.928	1.865	2.152	2.060	4.214
1875	4.484	2.036	1.933	2.069	2.300	2.068	1.915	1.860	2.147	2.328	4.156
1876	4.632	2.056	1.918	2.102	2.410	2.112	1.915	1.865	2.130	2.508	4.124
1877	4.437	2.036	1.947	2.032	2.338	2.129	1.928	1.885	2.113	2.408	4.029
1878	4.251	1.992	2.003	1.968	2.118	2.120	1.950	1.890	2.105	2.166	4.085
1879	4.187	1.955	2.073	1.931	1.825	2.080	1.990	1.941	2.066	1.861	4.326
1880	3.975	1.945	2.088	1.900	1.638	2.008	2.030	1.977	2.030	1.616	4.359
1881	3.881	1.973	2.041	1.949	1.600	1.930	2.063	2.023	1.988	1.567	4.314
1882	3.906	2.018	1.991	1.989	1.730	1.890	2.080	2.059	1.950	1.707	4.199
1883	4.434	2.051	1.933	2.058	1.930	1.870	2.088	2.090	1.920	1.940	4.494
1884	4.712	2.049	1.918	2.069	2.196	1.885	2.080	2.115	1.895	2.207	4.505
1885	4.870	2.015	1.947	2.102	2.345	1.925	2.063	2.135	1.876	2.411	4.459
1886	4.896	1.971	2.003	2.032	2.404	2.005	2.030	2.140	1.864	2.439	4.457
1887	4.530	1.945	2.073	1.968	2.262	2.083	1.990	2.135	1.858	2.314	4.216
1888	4.276	1.957	2.088	1.931	2.008	2.120	1.950	2.115	1.846	1.985	[illegible]
1889	4.062	1.998	2.041	1.900	1.728	2.130	1.928	2.090	1.842	1.657	4.405
1890	3.922	2.039	1.991	1.949	1.604	2.114	1.915	2.059	1.842	1.513	4.409
1891	3.938	2.056	1.933	1.989	1.643	2.060	1.915	2.023	1.846	1.465	4.473
1892	4.111	2.033	1.918	2.058	1.810	1.929	1.928	1.977	1.858	1.511	4.600
1893	4.308	1.990	1.947	2.069	2.042	1.917	1.950	1.941	1.864	1.720	4.588
1894	4.566	1.953	2.003	2.102	1.881	1.990	1.990	1.910	1.876	1.984	4.582
1895	4.751	1.946	2.073	2.032	2.397	1.871	2.030	1.885	1.895	2.129	4.622
1896	4.596	1.977	2.088	1.968	2.366	1.894	2.063	1.865	1.920	2.141	4.455
1897	4.428	2.022	2.041	1.931	2.176	1.942	2.080	1.860	1.950	2.002	4.426
1898	4.181	2.052	1.991	1.900	1.888	2.030	2.088	1.865	1.987	1.801	4.380
1899	3.651	2.047	1.933	1.949	1.662	2.094	2.080	1.885	2.032	1.686	3.965
1900	3.762	2.012	1.918	1.989	1.595	2.124	2.063	1.990	2.044	1.677	4.085
1901	3.960	1.968	1.947	2.058	1.700	2.128	2.030	1.941	2.095	1.867	4.093
1902	4.285	1.945	2.003	2.069	1.885	2.105	1.990	1.977	2.113	2.087	4.198
1903	4.558	1.960	2.073	2.102	2.145	2.060	1.950	2.023	2.130	2.443	4.115
1904	4.770	2.001	2.088	2.032	2.325	1.960	1.928	2.059	2.147	2.540	4.230
1905	4.789	2.041	2.041	1.968	2.410	1.908	1.915	2.090	2.152	2.525	4.264
1906	4.586	2.055	1.991	1.931	2.300	1.873	1.915	2.115	2.158	2.338	4.248
1907	3.987	2.031	1.993	1.900	2.062	1.873	1.928	2.135	2.160	2.022	3.965
1908	3.533	1.986	1.918	1.949	1.773	1.904	1.950	2.140	2.158	1.778	3.755
1909	3.577	1.951	1.947	1.989	1.620	1.959	1.990	2.135	2.152	1.723	3.854
1910	3.778	1.948	2.003	2.058	1.617	2.050	2.030	2.115	2.147	1.968	3.810
1911	4.079	1.980	2.073	2.069	1.766	2.105	2.063	2.090	2.130	2.276	3.803
1912	4.359	2.025	2.088	2.102	1.988	2.128	2.080	2.059	2.113	2.583	3.776
1913	4.491	2.053	2.041	2.032	2.232	2.125	2.088	2.023	2.105	2.699	3.792
1914	4.350	2.045	1.991	1.968	2.375	2.095	2.080	1.977	2.066	2.596	3.754
1915	4.140	2.008	1.933	1.931	2.390	2.010	2.063	1.941	2.030	2.306	3.834
1916	4.211	1.965	1.918	1.900	2.219	1.945	2.030	1.910	1.988	1.875	4.336
1917	3.759	1.944	1.947	1.942	1.942	1.899	1.990	1.885	1.950	1.507	4.252
1918	3.366	1.963	2.003	1.989	1.690	1.871	1.950	1.865	1.920	1.251	4.115
1919	3.229	2.004	2.073	2.058	1.598	1.878	1.928	1.860	1.895	1.294	3.935
1920	3.410	2.043	2.088	2.069	1.670	1.915	1.915	1.865	1.876	1.441	3.969
1921	3.663	2.054	2.041	2.102	1.845	1.980	1.915	1.885	1.864	1.586	4.077
1922	3.780	2.028	1.991	2.032	2.092	2.068	1.928	1.990	1.858	1.907	3.873
1923	4.029	1.983	1.933	1.968	2.300	2.112	1.950	1.941	1.846	2.033	3.996
1924	4.055	1.949	1.918	1.931	2.410	2.129	1.990	1.977	1.842	2.146	3.909
1925	4.207	1.949	1.947	1.900	2.338	2.120	2.030	2.023	1.842	2.149	4.068
1926	4.055	1.983	2.003	1.949	2.118	2.080	2.063	2.059	1.846	2.101	3.954
1927	3.927	2.028	2.073	1.989	1.825	2.008	2.080	2.090	1.858	1.951	3.976
1928	3.662	2.054	2.088	2.058	1.638	1.930	2.088	2.115	1.864	1.835	3.827
1929	3.479	2.043	2.041	2.069	1.600	1.890	2.080	2.135	1.876	1.734	3.745
1930	3.499	2.004	1.991	2.102	1.730	1.870	2.063	2.140	1.895	1.795	3.704
1931	3.524	1.963	1.933	2.032	1.930	1.885	2.030	2.135	1.920	1.828	3.696
1932	3.674	1.944	1.918	1.968	2.196	1.925	1.990	2.115	1.950	2.006	3.668
1933	3.822	1.965	1.947	1.931	2.348	2.005	1.950	2.090	1.987	2.223	3.599
1934	3.940	2.008	2.003	1.900	2.404	2.083	1.928	2.059	2.032	2.417	3.523
1935	3.948	2.045	2.073	1.949	2.262	2.120	1.915	2.023	2.044	2.453	3.495
1936	3.841	2.064	2.088	1.989	2.008	2.130	1.915	1.977	2.095	2.256	3.585
1937	3.620	2.025	2.041	2.058	1.728	2.114	1.928	1.941	2.113	1.948	3.672
1938	3.506	1.979	1.991	2.069	1.604	2.060	1.950	1.910	2.130	1.693	3.813
1939	3.471	1.948	1.933	2.102	1.643	1.929	1.990	1.885	2.147	1.577	3.894
1940	3.420	1.951	1.918	2.032	1.810	1.917	2.030	1.865	1.152	1.675	3.745
1941	3.449	1.986	1.947	1.968	2.042	1.881	2.063	1.860	2.158	1.906	3.543
1942	3.478	2.031	2.003	1.931	2.268	1.871	2.080	1.865	2.160	2.209	3.269
1943	3.606	2.055	2.073	1.900	2.397	1.894	2.088	1.885	2.158	2.450	3.156
1944	3.688	2.041	2.088	1.949	2.366	1.942	2.080	1.990	2.152	2.528	3.160
1945	3.567	2.001	2.041	1.989	2.176	2.030	2.063	1.941	2.147	2.388	3.179
1946	3.508	1.960	1.991	2.058	1.888	2.094	2.030	1.977	2.130	2.128	3.380
1947	3.416	1.944	1.933	2.069	1.662	2.124	1.990	2.023	2.113	1.858	3.558
1948	3.211	1.968	1.918	2.102	1.595	2.128	1.950	2.059	2.105	1.825	3.386
1949	3.169	2.012	1.947	2.032	1.700	2.105	1.928	2.090	2.066	1.880	3.289
1950	3.582	2.048	2.003	1.968	1.885	2.060	1.915	2.115	2.030	2.024	3.558
1951		2.053	*2.088	1.931	2.145	1.960	1.915	*2.115	*2.030	2.231	
1952		2.022	2.041	1.900	2.325	1.908	1.928	2.135	1.988	2.247	
1953		1.976	1.991	1.949	2.410	1.873	1.960	2.140	1.950	2.239	
1954		1.947	1.933	1.989	2.300	1.873	1.990	2.135	1.920	2.087	

BRIEF ARTICLES

CYCLES IN FLIGHT YEARS OF SNOWY OWL, EVENING GROSBEAK, AND PINE GROSBEAK

By Leonard W. Wing

Among the records possibly influenced by cycles are historical records of past events. Unless continuous figures are available, such records may not be analyzed by periodic tables or other numerical procedures. Dividing the length of the record by the number of happenings as the basis for determining the average interval is a crude method at best. The time chart is suggested here as a useful tool for studying rhythm in such records.

Many events of possible cyclic nature have been recorded. Among these are such things as river floods, auroras, earthquakes, seed years, insect outbreaks, or drought. The chronicles of the Chinese, for example, mention auroras, droughts, and earthquakes, in some cases over periods of many centuries. Historical accounts of drought and flood have been given. Many writings have told of insect outbreaks, crop failures, and seed years of forest trees. The invasions or flight years of northern birds have been noted for many decades. Those of the Snowy Owl, Pine Grosbeak, and Evening Grosbeak will be used here to illustrate the use of time charts in such a series. In a sense, we have only records for the "peak years," those between being missing or non-existent.

Snowy Owl flights about every four years are expected in Northern United States and Canada.* A four year time chart shows that the timing has been at almost precisely four years since 1833 (Figure 1). Reports in literature seem missing for the 1849, 1857, 1869, and 1873 intervals, but flights presumably took place. The 1841 interval came two years early, but others have come at most one year earlier or later than the expected time if the four year interval is the correct one. (It is possible that the cycle length is slightly shorter than four years, but a time chart will not show it with any greater precision). It is likely that in addition to the four-year cycle, other cycles are present. One of about 5.1-years in length seems a possibility.

Figure 2 shows a four-year time chart of the Pine Grosbeak in the Lake States. The record has gaps for the 1886, 1894, and 1902 interval, but reports for these years may be expected to be found in literature if the timing shown is correct. There seem to be suggestions of other cycle lengths; one of which may be slightly longer than five years.

An alternation of years of invasion of some northern birds has been suggested as a two-year cycle. Figure 3 shows a two-year time chart of the Evening Grosbeak in New England as reported in the Audubon Christmas Censuses. Only in the period 1916-1918, when the birds came three years in a row, has the two-year timing been "off". No flight seems to have been reported for 1937.

*GROSS, A. O., 1947. *Cyclic Invasions of the Snowy Owl and the Migration of 1945-1946.* AUK 64:584-601.

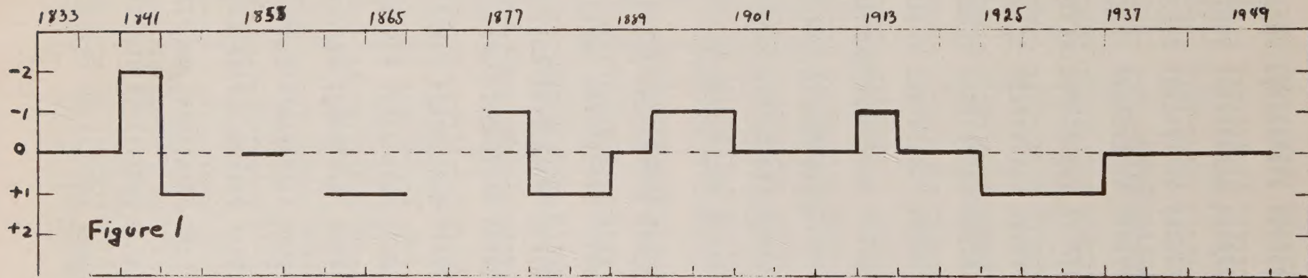


Figure 1

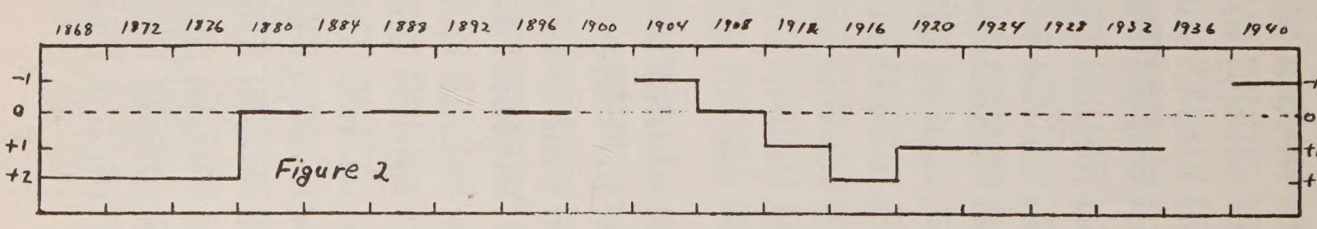


Figure 2

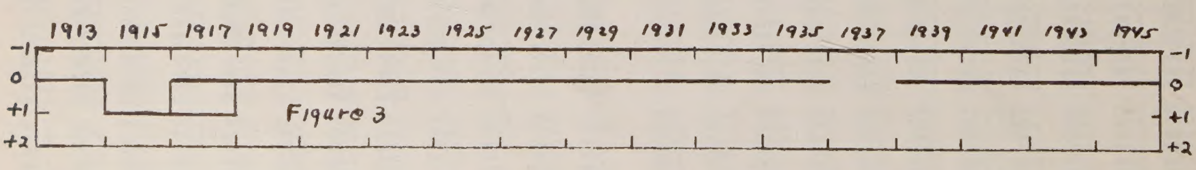


Figure 3